# Distance Distribution Analysis of Point-Objects

# and

# Geological Features


## Abstract:

Natural resource explorationists commonly encounter the need to know whether there is a systematic spatial association between the distribution of resource occurrences represented as points on a map, and some geological feature set such as a faults, igneous intrusive contacts, fold axes, etc., represented as points, polylines or polygons. Similar types of problems occur in a broad array of subject matter areas when the map distribution of multiple point occurrences of a phenomenon of interest relative to geo-objects represented by points, polylines, or polygons may indicate a dependent or conditional spatial relationship between the two feature sets.

The distance distribution tool described in this tutorial has been developed for use in the esri® ArcMap™ ArcGis geographic information system. It is derived from modifying the raster-based distance distribution algorithm reported by Carranza (2009) to function within the esri® ArcMap™ vector-based GIS. Carranza's discussion includes references that are helpful in understanding constraints on the appropriate use of the tool. Berman's (1986) publication is especially useful in outlining the physical scenario for which the output of the tool is valid. The comments of Marshall (1979) concerning perceived effectiveness of lineaments as a guide to mineral occurrences suggests ways in which distance distribution analysis might be useful in focusing exploration efforts.


## Introduction:

[ By using the distance distribution tool to process various subsets of features extracted from these files, the exercise demonstrates how one is able to assess the nature and significance of the spatial distribution between various sets of geo-objects (faults and igneous plutons) and the location of point-objects (mineral occurrence) displayed on a map (Fig. 1).

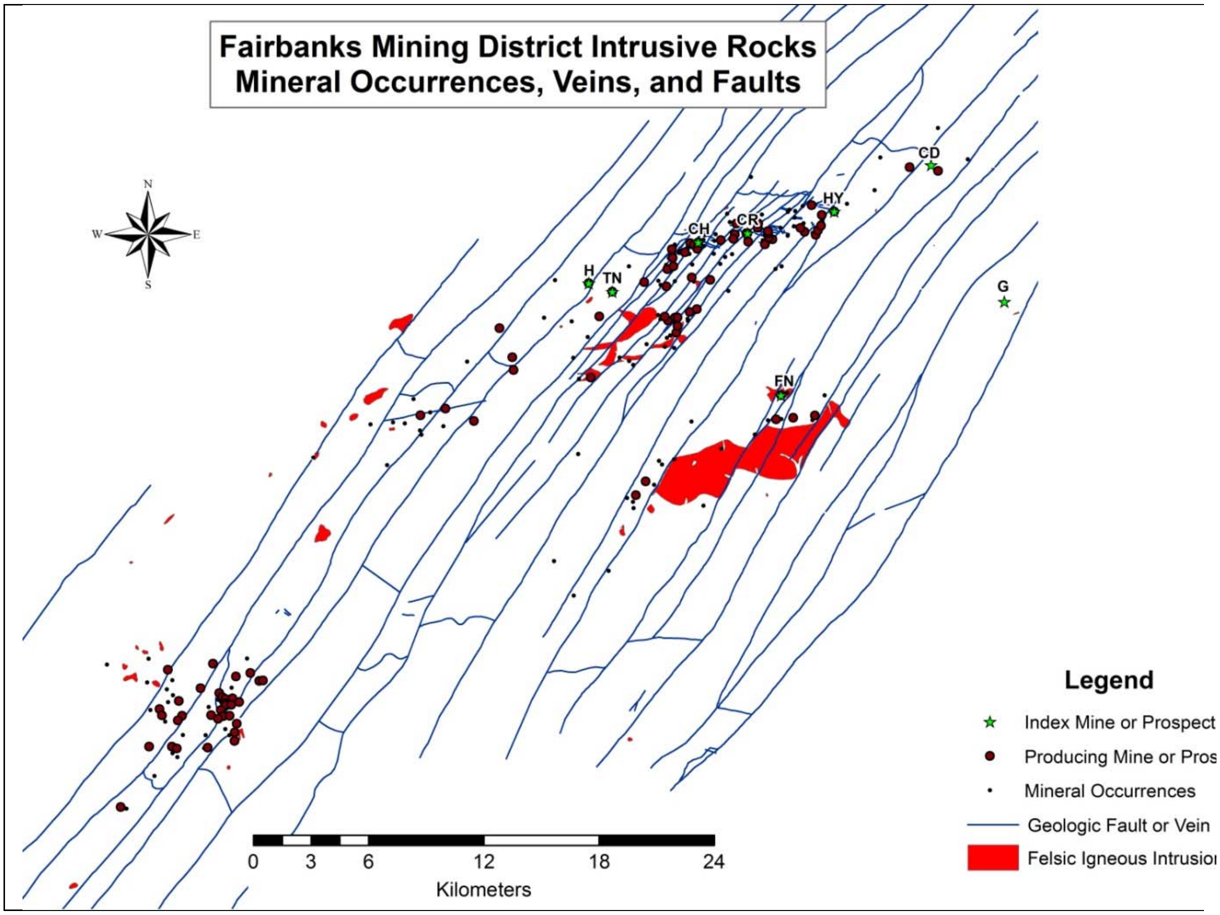


Figure 1. Map of point-objects (mineral occurrences, and producing mines or prospects) and geo-objects (district-scale faults, veins, and felsic igneous intrusions) in the Fairbanks Mining District, Alaska. Modified from, Newberry, et al. (1996) H = Hindenburg , TN = True North, CH =Cleary Hill , CR = Cristina, Hy = Hi-Yu, CD = Coffee Dome, G = Gil, and FN = Fort Knox .

| Shapefile | Features |
|---|---|
| FbkMinOccs.shp | 232 Fairbanks mineral occurrences of any type |
| ClryMinOccs.shp | 143 Cleary Summit mineral occurrences of any type |
| FbkProdOccs.shp | 94 Fairbanks District sites having recorded production |
| ClryProdOccs.shp | 51 Cleary Summit sites having recorded production |
| FBKFaults.shp | Regional geologic fault and vein feature set |
| FbxIgnInt.shp | Fairbanks igneous intrusive rocks |

Table 1. Set of shapefiles used in the Distance Distribution Analysis tool tutorial exercises.

The distance distribution analysis is performed by determining whether the point-objects of interest occur more frequently near the geo-objects of interest than would be expected to happen by chance. This determination can be made by comparing the cumulative frequency of the number of point-objects occurring within successively greater fixed distances from geo-objects of interest as compared to the cumulative number of random locations that are available within the same successively greater fixed distances. This information can be displayed on a distance distribution graph such as the one displayed below for the Cleary Summit area, Fairbanks Mining District, Alaska

(Fig. 2d.). The graph in Figure 2d is derived from the data generated by the Distance Distribution tool (Table 2). Brief definitions of the column headings displayed in Table 2 are listed in Table 3.

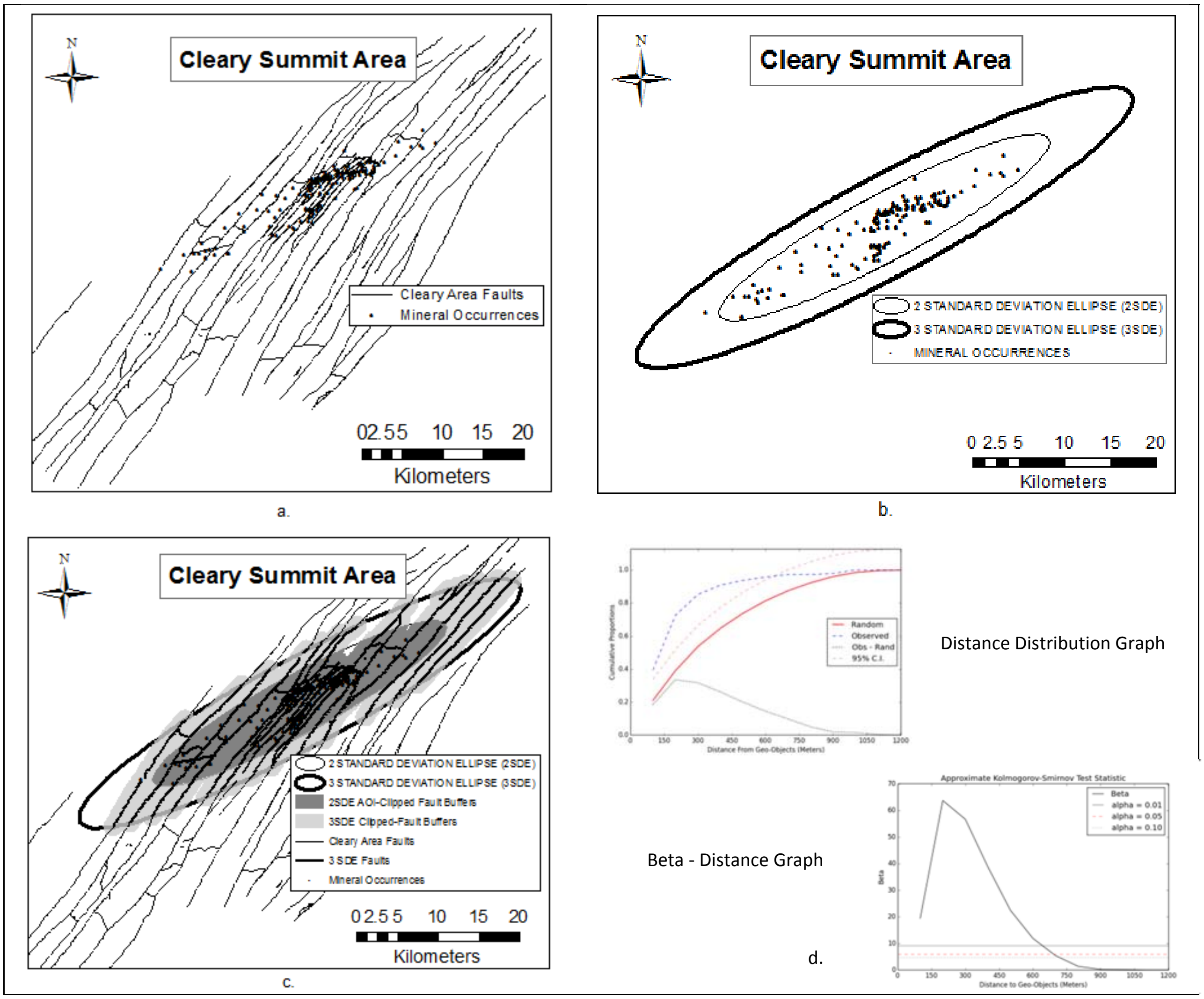


Figure 2. Conceptual milestones in the distance distribution analysis process: a. - input point- and geo-objects, b. - point-object spatial distribution ellipses used to constrain geo-object extents to the span of point-object extents, c.- geo-object buffers constrained to point-object Area of Interest (AOI), d.- Distance Distribution and Beta - Distance graphs output by the Distance Distribution Analysis tool.

| FID | Distance | Buff_Area | Cum_Area | Ex | NumDeps | CumNumDeps | Ox | Ox_EX | UpConfi | Beta |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 100 | 48204816 | 48204816 | 0.209 | 56 | 56 | 0.394 | 0.185 | 0.336749 | 19.43 |
| 1 | 200 | 41636530 | 89841346 | 0.390 | 47 | 103 | 0.725 | 0.335 | 0.517626 | 63.77 |
| 2 | 300 | 33565251 | 123406597 | 0.536 | 18 | 121 | 0.852 | 0.316 | 0.66344 | 56.72 |
| 3 | 400 | 25544333 | 148950930 | 0.647 | 8 | 129 | 0.908 | 0.261 | 0.77441 | 38.80 |
| 4 | 500 | 20875556 | 169826485 | 0.738 | 4 | 133 | 0.937 | 0.199 | 0.865097 | 22.46 |
| 5 | 600 | 17356557 | 187183042 | 0.813 | 3 | 136 | 0.958 | 0.145 | 0.940498 | 11.87 |
| 6 | 700 | 14063253 | 201246295 | 0.874 | 2 | 138 | 0.972 | 0.098 | 1.001591 | 5.408 |
| 7 | 800 | 11075882 | 212322177 | 0.922 | 0 | 138 | 0.972 | 0.049 | 1.049707 | 1.390 |
| 8 | 900 | 8678863 | 221001040 | 0.960 | 1 | 139 | 0.979 | 0.019 | 1.08741 | 0.201 |
| 9 | 1000 | 5712537 | 226713576 | 0.985 | 3 | 142 | 1 | 0.015 | 1.112226 | 0.130 |
| 10 | 1100 | 2600272 | 229313848 | 0.996 | 0 | 142 | 1 | 0.004 | 1.123522 | 0.008 |
| 11 | 1200 | 878155 | 230192004 | 1 | 0 | 142 | 1 | 0 | 1.127337 | 0 |

Table 2. - Data derived from the Distance Distribution Analysis tool. Where: FID = ArcMap Feature Identification Number for a multipart buffer surrounding the geo-objects of interest, Distance = Distance to the outer limit of a ring buffer surrounding a geo-object , Buff_Area = Area within a single of ring buffer, Cum_Area = The cumulative area of all ring buffers, summed from the geo-object boundary to outer boundary of the identified ring buffer, Ex = Expected cumulative proportion of Random locations within the ring buffers, summed from the geo-object boundary to the outer boundary of the identified ring buffer, Num_Deps = Number of point object locations (mine locations in this example) within the identified ring buffer, CumNumDeps = The cumulative number of point object locations within the identified ring buffers, summed from the geo-object boundary to the outer limit of the identified ring buffer, Ox = Cumulative proportion of Observed point object locations, summed from the geo-object boundary to the outer limit of the identified ring buffer, Ox_Ex = Difference between the Observed vs. the Expected cumulative proportion of point objects locations contained within the identified ring buffer, UpConfi = Upper 95% confidence limit for the expected cumulative proportion of random locations within identified ring buffers, summed from the geo-object boundary to the outer limit of the identified buffer, Beta = The Kolmogorov-Smirnov test statistic for the comparison of the observed vs. expected cumulative proportion of observed vs. expected point object locations.

| | |
|---|---|
| FID | ArcMap Feature Identification Number for a multipart buffer surrounding the geo-objects of interest |
| Distance | Distance from the boundary of a geo-object to the outer limit of an identified ring buffer surrounding the geo-object boundary |
| Buff_Area | Area within a single identified ring buffer |
| Cum_Area | The cumulative area of successive ring buffers, summed from the geo-object boundary to the outer limit of the identified buffer |
| Ex | Expected cumulative proportion of random locations included within successive ring buffers, summed from the geo-object boundary to the outer limit of the identified buffer |
| Num_Deps | Number of point object locations (mineral occurrences) within a ring buffer |
| CumNumDeps | The cumulative number of point object locations within the identified ring buffers, summed from the geo-object boundary to the outer limit of the identified ring buffer |
| Ox | Ox = Cumulative proportion of Observed point object locations, summed from the geo-object boundary to the outer limit of the identified ring buffer |
| Ox_Ex | Difference between the Observed vs. the Expected cumulative proportion of point objects locations contained within the identified ring buffer |

| | |
|---|---|
| UpConfi | Upper 99% confidence limit for the expected cumulative proportion of random locations within identified ring buffers, summed from the geo-object boundary to the outer limit of the identified buffer |
| Beta | The Kolmogorov-Smirnov test statistic for the comparison of the observed vs. expected cumulative proportion of observed vs. expected point object locations |

Table 3. Definition of the variables generated by the Distance Distribution Analysis too.

In order to conduct a valid distance distribution analysis, it is necessary that the spatial domain of the point-objects span the spatial domain of the geo-objects of interest, Berman (1986). To ensure this criteria is met, the Distance Distribution Analysis (DDA) tool requires the user to define an Area of Interest (AOI) that limits the analysis to an area constrained by a 1-, 2- , or 3-standard deviation spatial ellipse (SDE) derived from the directional distribution of the point-objects of interest; and optionally allows the user to add an additional constraining geo-barrier prior to executing the tool's cumulative frequency calculations.

Key milestones in the process of defining a valid analysis area are illustrated in Figure 2a - c. The user first selects the point- and geo-objects to be analyzed. Figure 2a shows that for Exercise 1, the regional faults that transect the Cleary Summit area and 143 known mineral occurrences that are more closely confined to the Cleary Summit area comprise the geo-objects and point-objects, respectively that will be assessed for the character and significance of their spatial association. Inspection of the map (Fig. 2a) reveals that the initial data are not in compliance with Berman's (1986) criteria of co-spanning spatial distributions of the point- and geo-objects of interest. The regional faults extend beyond the limit of known mineral occurrence locations. Given this situation, use is made of the map of mineral occurrence locations to construct a three standard deviation (3SDE) directional distribution elliptical polygon that is, based on the initial data, inferred to encompass 99% of all expected mineral occurrences in the Cleary Summit area if the currently known mineral occurrences have a spatially normal distribution (Fig 2b). This 3SDE polygon is used to extract the segments of faults within the 99% spatial domain of the mapped mineral occurrence locations (Fig. 2c).

At this point in the analysis process, the extracted fault segments (Fig. 2c) still do not conform to Berman's (1986) criteria; but by eliminating large extents of fault traces beyond the 3SDE ellipse, much processing time is saved in constructing the ring buffers around that portion of the fault set that ultimately will be considered in the distance distribution assessment. After geo-object buffers have been generated for geo-object segments within the area of the 3SDE directional distribution ellipse, by default, they are clipped with the point-object's two standard deviation (2SDE) directional distribution ellipse polygon (Fig. 2b). A two standard deviation directional distribution ellipse will encompass 95% of all expected mineral occurrences in the Cleary Summit area if the currently known mineral occurrences are spatially normally distributed.

The use of the point-object’s 2SDE polygon as an extraction mask removes the remaining portion of the geo-object ring buffers that extend beyond the span of point-object locations (Fig. 2c). If additional pruning of the geo-object buffers is required for the spatial domains of point- and geo-objects to coincide for assessment, the DDA tool provides for the inclusion of an additional optional user-defined geo-barrier to impose a final constraint on the assessment area of interest. The use of directional distribution ellipses to constrain the distance - distribution analysis AOI occasionally omits some outlying point-objects, however, the preponderance of point objects are retained while helping to enforce coincidence of point-object and geo-object analysis domains.

The final output of the DDA tool consists of two graphs (Fig. 2d) that visually summarize the data compiled in Table 1. Table 1 data are accessible in a .csv file that bears the name “DistanceDistribution.csv" and also in the attribute table of the AOI-clipped geo-object buffer file having a name that is the same as the basename of the input geo-object file plus the suffix "_2 _STANDARD_DEVIATIONS_AOI_BUFFERS.shp."

## Tutorial Exercises:

The Distance Distribution Analysis ver 2 **tool** requires the esri® ArcMap™ Spatial Statistics extension and functions in ArcGis® Desktop v. 10.1 and 10.2. The Python script for the Distance Distribution Analysis ver 2 tool was written with Python version 2.7x which is included with ArcGis® v. 10.x The Distance Distribution Analysis ver 2 tool was developed on a desktop computer having a 64-bit Windows 7© operating system.

Five tutorial exercises are provided to illustrate how to use the Distance Distribution Analysis ver 2 tool to generate data that can be used to assess the character and significance of spatial association between objects mapped as points and geologic features mapped as points, polylines, or polygons.

For this tutorial, the shapefiles listed in Table 1, above, are provided in the "***SourceData***" subfolder of the MS "***DistanceDistributionAnalysis ver 2 Tutorial*"** folder. The Distance Distribution Analysis ver 2 tool is found in the ***Distance Distribution Analysis ver 2 Tutorial>DistanceDistributionToolbox> DistanceDistributionTool.tbx*** toolbox. The Distance Distribution Analysis tool is a "script" tool and will be displayed as such when it appears in the ArcMap™ Catalog window.

### Exercise 1:

**<u>Installing the Distance Distribution Analysis Tool</u>:**

This tutorial requires the file structure and files contained in the ***Distance Distribution*** **Analysis ver 2** ***Tutorial*** folder (**Distance** ***Distribution Toolbox***, ***Documentation, Exercises,*** and ***SourceData***).

Installing the **Distance Distribution Analysis.tbx** and the tutorial data files is accomplished by following the workflow outlined below:

1. From the disk or download file provided, copy the ***Distance Distribution*** **Analysis ver 2** ***Tutorial*** folder and subfolders to any of your disk drives that can be accessed by ArcMap™. The ***Distance Distribution*** **Analysis ver 2** ***Tutorial*** folder contains subfolders named:
    - **Distance Distribution Analysis Toolbox,**
    - **Documentation,**

- **Exercises**
  - **Exercise_1**
  - **Exercise_2**
  - **Exercise_3**
  - **Exercise_4**
  - **Exercise_5**
- **Source Data.**

These folder names can be given different user-preferred names, if desired.

2. Link the Python script, *"DistanceDistribution_ver_2_09192014.py*," found in the **Distance Distribution Analysis Toolbox** folder to the Distance Distribution Analysis ver 2 script tool. This is accomplished in ArcMap™ by browsing to the Distance Distribution Analysis ver 2 tool in the Catalog window of ArcMap™ ( ***Distance Distribution Analysis ver 2 Tutorial > Distance Distribution*****Analysis*****Toolbox*** > Distance Distribution Analysis ver 2**).** Right-click the Distance Distribution Analysis ver 2 ***script*** tool. Select Properties>Source, and browse to the *DistanceDistribution_ver_2_09192014.py* file located in the **Distance Distribution Analysis Toolbox** folder, select that file and then Select "OK" at the bottom of the Distance Distribution Analysis ver 2 tool's Properties dialog box.

<u>Applying the Distance Distribution Analysis Tool</u>:

- Set the ArcMap™ session Environment Workspace to: ***<Drive>:\<Path>\DistanceDistributionAnalysis\Exercise_1***
- Double-click the Distance Distribution Analysis ver 2 tool to open it. The following tool dialog window will appear (Fig. 4). The dialog window prompts for 7 user-specified tool parameters.

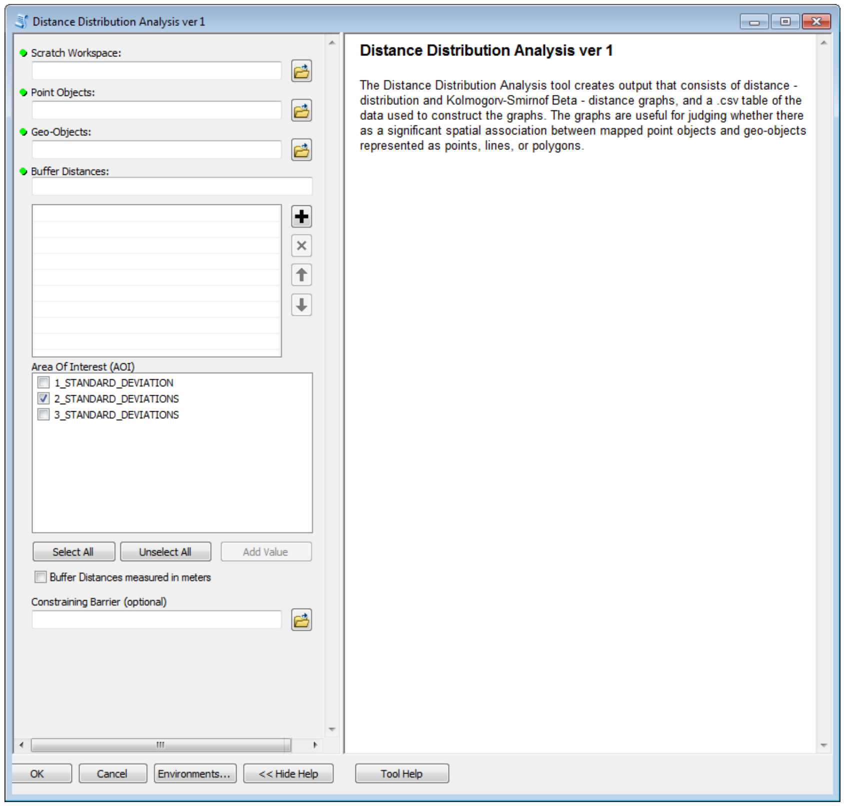


Figure 4. Distance Distribution Analysis tool input parameter dialog interface window.

- <u>*Scratch Workspace*</u>: The Distance Distribution Analysis tool generates several output files. These files will be written to the workspace identified in the Scratch Workspace parameter window. For this tutorial, in the <u>*Scratch Workspace*</u> parameter window; browse to: ***Drive>:\<Path>\DistanceDistributionAnalysis\Exercise_1****;* and click "Add."

- <u>*Point Objects*</u>: For this tutorial, the *Point-Object .shp* files are located in the **SourceData** subfolder. In the <u>*Point Objects*</u> parameter window, browse to the file folder that contains the point objects '-.shp' file of interest for the distance distribution analysis and 'Add' the point file to the <u>*Point-Object*</u> parameter dialog window. For this exercise choose the *ClryMinOccs.shp* file.

- <u>*Geo-Objects*:</u> For this tutorial, the *Geo-Object .shp* files are located in the **SourceData** file folder. In the <u>*Geo-Objects*</u> parameter window, browse to the file folder that contains the geo-objects '-.shp' file of interest for the distribution analysis and 'Add' the geo-object file to the <u>*Geo-Object*</u> parameter dialog window. For this exercise choose the *FbkFaults.shp* file.

- <u>*Buffer Distances*</u>: By default, buffer distances are expressed in kilometers. Individual buffer rings are identified by the distance from the geo-object to the outer limit of the buffer ring. The <u>*Buffer Distance*</u> parameter window accepts a sequence of numbers (one entry at a time) that are added to a list that defines the outer distance limit of each buffer. The tool will generate one multipart buffer for each distance entry. Distances should be entered in order of increasing distance. (See *Buffer Distances Measured in meters* check-button if ring buffers measured in meters are desired). For this exercise, enter distances from 100 to 1200 meters in 100 meter increments into the *Buffer Distances* multiple-parameter window.

- *Area of Interest (AOI)*: By default, the distance distribution analysis Area of Interest is defined as the area within a 2 standard deviation distribution ellipse calculated from the directional distribution of the point-objects of interest. The AOI may be changed to a one- or three-standard deviation ellipse area by unselecting the default 2_STANDARD_DEVIATION setting and then selecting the standard ellipse extent of choice. For this exercise accept the default setting for the *Area of Interest* parameter window.

- *Buffer Distances measured in meters*: Checking the *Buffer Distances measured in meters* parameter check box causes the Distance Distribution Analysis tool to treat all values entered in the *Buffer Distances* parameter window as distances expressed in meters. For this exercise select the *Buffer Distances measured in meters* check box to measure distances in meters.

- *Constraining Barrier* (optional): The *Constraining Barrier* parameter window allows the user to additionally constrain the Area of Interest to the intersection of the AOI-ellipse area and any existing user-created polygon '-.shp' file. If an additional constraining geo-barrier is desired, browse to the desired constraining barrier .shp file and 'Add' it to the *Constraining Barrier* parameter window. For this exercise, do not enter a constraining geo-barrier.

When all indicated parameter entries have been made, select 'OK' to launch the Distance Distribution Analysis tool.

All results generated by the Distance Distribution Analysis tool will be sent to the file folder named ***'Exercise_1'*** that was entered in the *Scratch Workspace* parameter window. After running the tool, it may be necessary to 'Refresh' the chosen Scratch Workspace in the ArcMap™ catalog window in order to make the Distance Distribution and Beta Distribution graph image icons appear in the ArcMap™ Catalog window.

Henceforth in this tutorial the Distance Distribution Analysis too will be referred to as the "DDA tool."

Additional Parameter Notes:

Scratch Workspace: refers to the full path specification that includes the folder name in which the user would like to store the Intermediate files generated by the DDA tool and the tools graphical output. It generally is convenient to store the generated files in a folder within the current project folder. If a scratch workspace folder already exists, the 'Browse' button located to the right of the *Scratch Workspace* parameter window can be used to navigate to the existing project scratch workspace folder, and select it. All DDA tool products will be sent to whichever folder is selected.

Area of Interest (AOI): It is important that the distance distribution analysis area of interest be confined to a region that closely conforms to Berman's (1986) criteria, i.e., the spatial domains of point-object and geo-objects should closely coincide. Violation of this condition will lead to biased results. For example, including large buffer areas that extend beyond the rational limits within which point-objects of interest can be expected to occur leads to a spurious inflation of the significance of any spatial association between point-and geo-objects. Use of a 2SDE directional distribution ellipse boundary for a set of point-objects provides a useful approximation of the point-object set's spatial domain.

Constraining Barrier (optional): Point-objects sometimes occur in geologic settings that preclude a reasonable extension of their domain into areas beyond well defined barrier features (for example a major fault or tectonic terrane boundary). In these instances, using only a directional distribution ellipse AOI polygon may not sufficiently constrain the geo-object ring buffers to prevent introducing an apparent, but spurious, inflation of the significance of any spatial association between the point- and geo-objects of interest. Such an outcome can be avoided by imposing an additional constraint on the geo-object ring buffers by employing a polygon feature-clipping mask consisting of the intersection of the selected AOI ellipse and a geo-barrier polygon created by the user prior to operating the DDA tool. A use of the 'Constraining Barrier' option will be demonstrated later in this tutorial.

Discussion of Output Products:

The DDA tool generates the following final outputs (Table 4):

| | |
|---|---|
| DistanceDistributionFigure.png | |
| BetaDistributionFigure.png | |
| DistanceDistribution.csv | |
| *ClryMinOccs_2_STANDARD_DEVIATIONS_AOI_Ellipse.shp* | |
| *ClryMinOccs_3_Sigma_Ellipse.shp* | |
| *FbkFaults_2_STANDARD_DEVIATIONS_AOI_Buffers.shp* | |
| *FbkFaults_3_Sigma_Clip.shp* | |
| *FbkFaults_Interm_Buffs.shp* | |

Table 4. Products generated by the Distance Distribution Analysis tool and place in the folder specified in the tool, 'Scratch Workspace' parameter window.

These files and images are stored in the folder that was previously selected in the DDA tool's *Scratch Workspace* parameter window.

Distance Distribution Graph:

Using the DDA tool with the previously indicated input files and other parameter settings results in the Distance Distribution graph shown in Figure 2d and Figure 4, below.

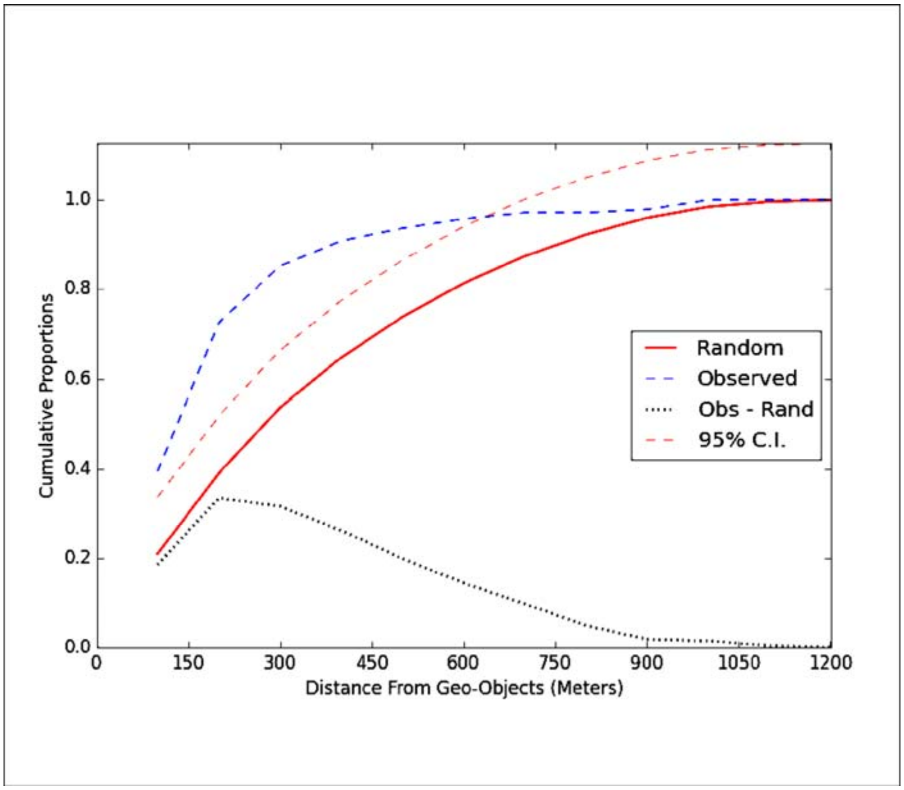


Figure 4. Distance distribution graph constructed from cumulative frequency measurements derived from the spatial location of mineral occurrences in the Cleary Summit area of interest in the Fairbanks Mining District, Alaska vs. regional faults present in that area of interest.

In Figure 4, the solid red line (Random) is the cumulative frequency curve for the total number of possible mineral occurrence locations available in the Cleary Summit AOI, summed at the outer ring-buffer boundary of successive ring buffers around the geo-objects of interest. This line is equivalent to plotting the cumulative frequency curve of a very large number of randomly located mineral occurrences within the Cleary Summit AOI.

The dotted red line is the upper 99% confidence limit for the cumulative frequency curve that would result if mineral occurrences (point-objects) on Cleary Summit were randomly located with respect to mapped faults (geo-objects). There is only a 1% chance that a large set of random point-objects in the Cleary Summit AOI would generate a cumulative frequency curve that would plot beyond this limit.

The blue dashed line in Fig. 4 is the cumulative frequency curve of actual observed mineral occurrence locations in the Cleary Summit area, summed at the outer ring-buffer boundary of successive ring buffers constructed around the fault segments located within Cleary Summit AOI. If there is a positive spatial association of the observed mineral occurrence locations with the faults in the Cleary Summit AOI, the 'Observed' cumulative frequency curve will plot above the 'Random' cumulative frequency curve. That is, positively spatially associated mineral occurrence locations will be more abundant near faults in the Cleary Summit AOI than they would be if they occurred randomly with respect to faults. If there is a negative spatial relationship of the observed mineral occurrence locations with respect to the faults in the Cleary Summit AOI, the 'Observed' cumulative frequency curve will plot below the 'Random' cumulative frequency curve. That is, if observed mineral occurrence locations are negatively spatially associated with faults they will occur less frequently near faults than they would if they occurred randomly with respect to faults.

If the 'Observed' cumulative frequency curve does not exceed the 99% confidence limit of the 'Random' cumulative frequency curve, one cannot be confident, at a 99% level of confidence, that any apparent spatial association displayed on the Distance Distribution graph has not happened by chance. The data plotted (Figure 4) indicate with that if the observed cumulative frequency curve of actual mineral occurrences in the Cleary Summit AOI is truly representative, there is less than a 1% chance that the positive spatial association between faults and mineral occurrences in the Cleary Summit AOI happened by chance.

The black dotted line (Obs - Rand; Fig. 4) represents the arithmetic difference between the value of the cumulative frequency curve for the mineral occurrence locations observed in successive geo-object ring-buffers in the Cleary Summit AOI vs. the value of the cumulative frequency curve for random locations in successive geo-object ring buffers in the Cleary Summit AOI.

Applying an interpretation similar to that of Carranza (2009): based on historical data and using a 99% confidence limit, the maximum value of the Beta curve (Fig. 5), and values compiled in Table 2, indicates the "optimal" distance from Cleary Summit faults for encountering mineral occurrences is within 200 meters. The Distance - Distribution plot (Fig. 4) indicates that within 200 meters of faults in the Cleary Summit AOI, one finds 73% of the mineral occurrences. The peak height of the Obs - Rand curve indicates that, for the Cleary Summit AOI, within 200 meters of Cleary Summit faults there is a 30% higher chance that a mineral deposit of some kind will occur than would be expected due to chance.

Beta - Distance Graph:

The DDA tool also generates a graph of the Kolmogorov-Smirnov test statistic (Beta) vs. Distance (Fig. 5). The Beta (β) statistic is used to test the Null Hypothesis that there is no significant difference between two cumulative frequency curves (Carranza, 2009; Goodman, 1954; Siegel, 1956) Because the Beta statistic value calculated for the distance - distribution assessment cumulative frequency curves in the Cleary Summit AOI exceeds the $\alpha = .01$ significance level, the test indicates that the positive spatial association exhibited in Figure 4 between observed mineral occurrences and faults within the Cleary Summit AOI is statistically significant at the α = .01 significance level. Figure 5 also more clearly displays the ring-buffer interval within which there is a significantly higher concentration of point-objects with respect to faults within the Cleary Summit AOI than would be expected to occur by chance. The α = .05 and α = 0.1 statistical significance levels also are displayer in this plot for comparison. Together, the outcomes displayed in Figures 4 and 5 indicate that there is a statistically significant positive spatial association between observed mineral occurrences and faults in the Cleary Summit AOI and they provide information about the distance range within which the positive spatial association is most pronounced.

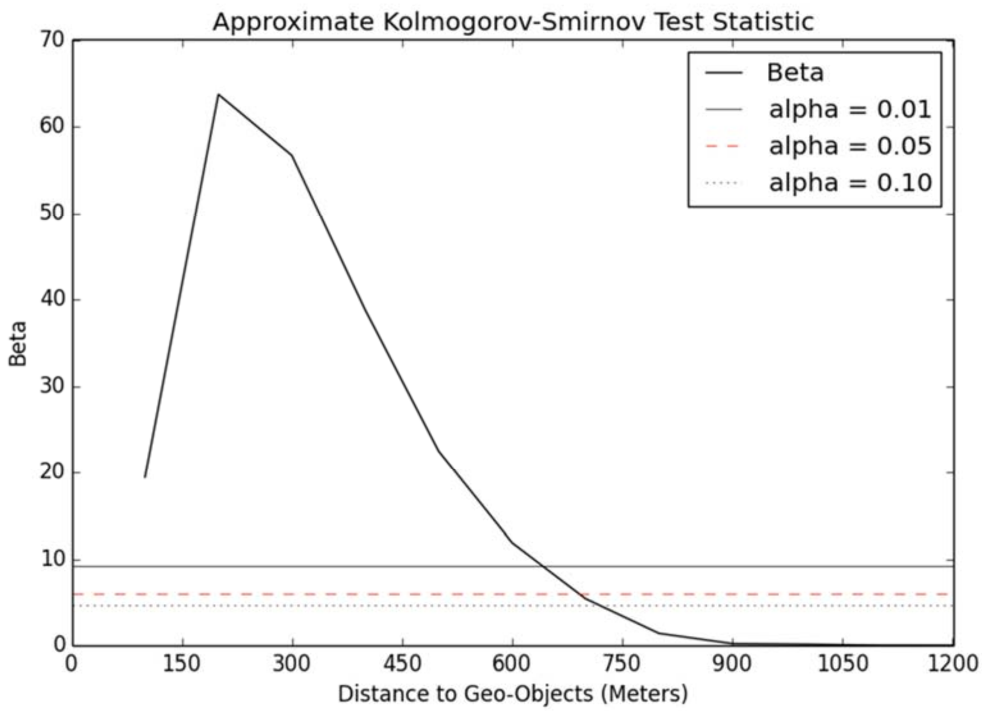


Figure 5. Plot of the approximate Kolmogorov- Smirnov test statistic (Beta) vs. distance from faults within the Cleary Summit distance - distribution area of interest (AOI).

Save the files and images created by the DDA tool in Exercise 1.

## Exercise 2:

The results derived in Exercise 1 are interesting, but in view of the abundance of closely space faults in the Cleary Summit AOI, one may question whether they only reflect the accidental circumstance of many random point-objects placed in an area in which there are so many closely spaced faults that it would be unlikely to find a mineral occurrence location that is more than 200 meters away from one of them (Fig. 6).

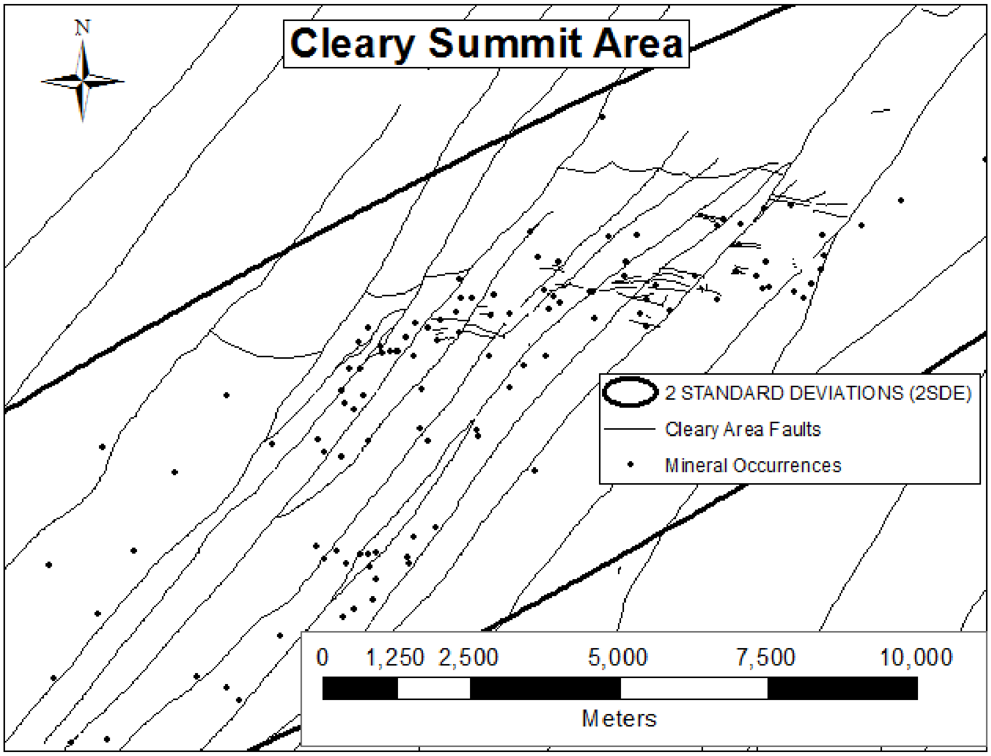


Figure 6. Cleary Summit Area fault and mineral occurrences

The DDA tool can be used to test whether such a criticism has merit. One can use ArcMap tools to generate 143 random point-objects within the 2SDE Cleary Summit AOI. Subjecting these 143 random point-objects and faults in the exact same Cleary Summit AOI to distance - distribution analysis will generate a new set of cumulative frequency data and plots that can be interpreted in the same manner as were the real world data of Exercise 1. If the distance - distribution of random points generates an apparently significant positive spatial association with faults in the Cleary Summit AOI, this would indicate that in spite of a significant statistical outcome for Exercise 1, it does not support much consideration with regard to mineral exploration in the Cleary Summit area.

The ArcMap™ **Data Management>Feature Class>Create Random Points** tool, will generate a user-specified number of random points in a user-specified polygonal area. It is therefore possible to use the 2SDE AOI ellipse generated in Exercise 1 to provide an identical area in which to create 142 random point-object locations. (In Exercise 1, one mineral occurrence lies outside the 2SDE AOI ellipse.)This set of 142 random points can then be substituted for the mineral occurrence locations analyzed in Exercise 1.

In order to ensure that the distance - distribution AOI in Exercise 2 is identical to the distance - distribution AOI of Exercise 1, it also will be necessary to use the Exercise 1 2SDE AOI ellipse as a constraining barrier in the analysis. When a constraining barrier is employed in the distance distribution analysis, the cumulative frequency data is derived from a DDA tool-generated shape file that has the following suffix: <geo-object basename> + "_2_STANDARD_DEVIATIONS_BC_AOI_Buffers.shp".

1. Open the Arc Toolbox catalog window and activate the Data Management>Feature Class> **Create Random Points** tool (Fig. 7).

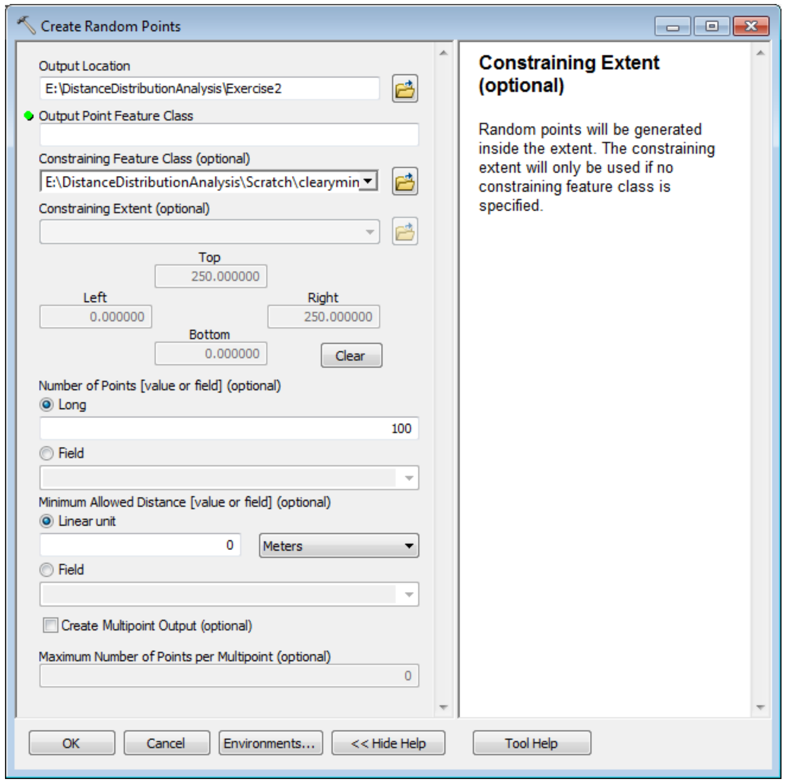

Figure 7. Arc Toolbox Data Management **Create Random Points** tool.

2. In the *Output Location* parameter window browse to the tutorials **Exercise_2** folder and Select “Add”.
3. In the *Output Point Feature Class* parameter window, enter "**ClearyRandomPoints**".
4. In the *Constraining Feature Class (optional)* parameter window, browse to the 2SDE ellipse shapefile created by the DDA tool in Exercise1, **ClryMinOccs_2_STANDARD_DEVIATIONS_AOI_ Ellipse.shp,** and **"ADD"** it to the parameter window.
5. In the *Number of Points [value or field] (optional)* parameter window, accept the default "Long" radio button and enter the integer "**142**," which is the number of mineral occurrences included in the 2SDE AOI ellipse created in Exercise 1.
6. In the Minimum *Allowed Distance [value or field] (optional)* parameter window, accept the default "Linear unit" radio button, and enter the integer "**50**." Accept the default unit designation, "Meters."
7. Click "**OK**" at the bottom of the Create Random Points dialog window. The tool will create a set of 142 point objects that are completely contained within the boundary of the 2SDE Cleary Summit AOI ellipse analyzed in Exercise 1 and will place them in the same folder as the tutorials other source data.

To complete Exercise 2, activate the **Distance Distribution Analysis** tool and enter the following parameters in the appropriate parameter windows.

8. In the _Scratch Worskspace_: parameter window, browse to the newly created **Exercise_2** folder, and "Add" it.
9. In the _Point Objects_: parameter window, browse to the _ClearyRandomPoints.shp_ file created with the Arc Toolbox - **Create Random Points** tool.
10. In the _Geo-Objects_: parameter window, browse to the _FbkFaults.shp_ file in the tutorial's **SourceData** folder and "Add" it.
11. In the _Buffer Distances_: multiple entries parameter window, enter the same sequence of buffer distances that were used in Exercise 1.
12. In the Area of Interest (AOI) multiple selection parameter window, accept the default "2_STANDARD_DEVIATIONS" choice.
13. Select the _Buffer Distances measured in meters_ parameter checkbox.
14. In the _Constraining Barrier (optional)_ parameter window, browse to the **Exercise_1** folder and "Add" the _ClryMinOccs_2_STANDARD_DEVIATIONS_AOI_Ellipse.shp_ file that was created by the **Distance Distribution Analysis** tool in Exercise 1.

The Distance Distribution and Beta Distribution graphs generated by the DDA tool for 142 random points relative to faults within the Cleary Summit AOI are shown in Figures 8, and 9.

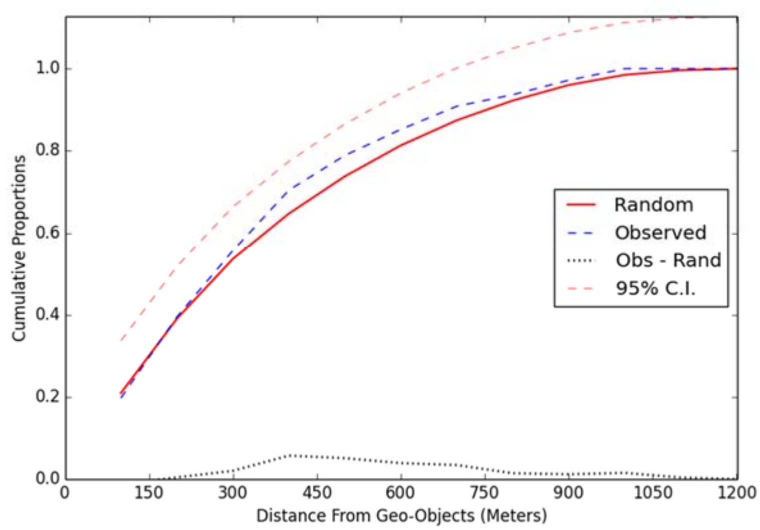


Figure 8. . Distance distribution plot constructed from cumulative frequency measurements derived from the spatial location of 142 random points within the Cleary Summit AOI in the Fairbanks Mining District, Alaska vs. regional faults present in the same area of interest.

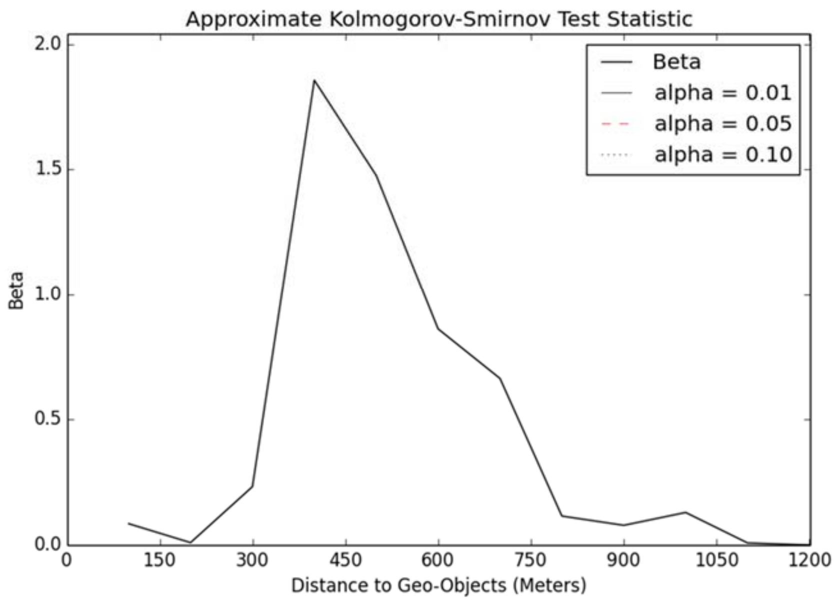


Figure 9. Plot of the approximate Kolmogorov- Smirnov test statistic (Beta) vs. distance from faults generated from random-point and fault locations within the Cleary Summit AOI.

Interpreting the plots shown in Figures 8 and 9 in the same manner as was done for the Distance - Distribution and Beta - Distance graphs for the actual mineral occurrences observed in the Cleary Summit AOI leads one to conclude that there is not a significant spatial association of 142 random point objects with faults within the Cleary Summit AOI. It, therefore seems unlikely that the strong spatial association indicated by the Distance - Distribution and Beta - Distance graphs generated by the DDA tool for the observed mineral occurrences relative to faults in the Cleary Summit AOI is an artifact of the abundance of mineral occurrences and close spacing of faults.

The outcome of Exercise 2 supports an inference that Cleary Summit faults played some role in constraining the present location of observed Cleary Summit mineralization. Inspection of Figure 1, however, indicates that even though the positive spatial association of mineral occurrences and faults in the Cleary Summit AOI is both statistically significant and physically meaningful; it is not, by itself, particularly useful in guiding future mineral exploration. There has been active mineral exploration in the Fairbanks Mining District for about 100 years, yet there are hundreds of miles of faults in the Fairbanks mining district with that have no indication of mineralization. Other factors, therefore, also must influence the location of mineralization.

## Exercise 3:

Within the Fairbanks mining district there are many igneous intrusions (Fig. 1) that represent a range of intrusive sizes, chemical compositions and geologic ages. Inspection of in Figure 1 suggests that Fairbanks mining district's known mineral occurrences are clustered and that they are more numerous in the general vicinity of an igneous body of some sort than they are elsewhere.

Exercise 3 tests that conjecture. In Exercise 3 the locations of 232 known Fairbanks lode mineral occurrences will be used.

There are a number of strategies that could be employed to subjectively impose an approximation of Berman's (1986) distance - distribution analysis requirement of coincident spans of point- and geo-object domains in the area of analysis. Before doing that, however, it is instructive to treat the entire set of intrusive geo-object and mineral occurrence point-object data sets as a whole in order to see where, or if, Berman's criteria is violated. To generate the cumulative frequency plots and statistics needed for the test, enter the following parameters in the DDA tool.

1. In the *<u>Scratch Worskspace</u>*: parameter window, browse to the newly created **Exercise_3** folder, and "Add" it.
2. In the *<u>Point Objects</u>*: parameter window, browse to the *FbkMinOccs.shp* file in the **SourcData** folder, and "Add" it.
3. In *<u>the</u>* *<u>Geo-Objects</u>*: parameter window, browse to the *FbkIgnInt.shp* file in the tutorial's **SourceData** folder and "Add" it.
4. In the *<u>Buffer Distances</u>*: multiple entries parameter window; enter a sequence of distances in units of 0.5 kilometers, from 0.5 to 10 kilometers.
5. In the *Area of Interest (AOI)* multiple selection parameter window, accept the default "2_STANDARD_DEVIATIONS" choice.
6. Do not select the *<u>Buffer Distances measured in meters</u>* parameter checkbox.
7. Click "OK" to run the DDA tool.

The DDA tool returns the Distance - Distribution and Beta - Distance graphs shown in Figure 10 a. and 10 b.

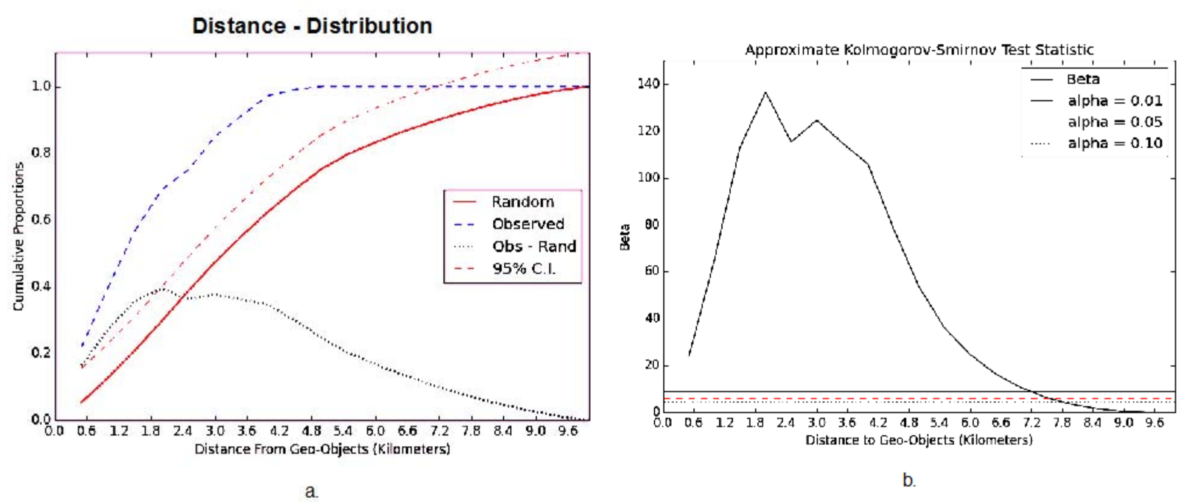


Figure 10 a. Distance -Distribution graph for 220 mineral occurrences in the Fairbanks Mining District relative to igneous intrusion ring buffers within the mineral occurrences' multi-part AOI; and b. Kolmogorov - Sminov Beta statistic for a sequence of outer igneous intrusion ring buffer distances.

Note that in Figure 10 a., all mineral occurrences falling within the 2SDE AOI entered in the DDA tool have been encountered within 5 kilometers of any intrusive, but the geo-object buffers persist out to 10 kilometers. This is an indication that the required condition that the point-objects AOI span the geo-objects of interest, has not been met. The result is that the significance of the positive spatial association between the Fairbanks Mining District mineral occurrences and the district's igneous intrusives is inflated in the graphs of Figure 1, and the indicated distance range of statistically significant positive spatial association has been inappropriately extended.

Examination of the map of the area of all Fairbanks District Mining District igneous intrusive rock buffers within 2SDE spatial distribution ellipse of all Fairbanks Mining District mineral occurrences visually confirms a poor coincidence between clusters of mineral occurrences and the areal extent of the trial area's geo-object ring buffers (Fig. 11).

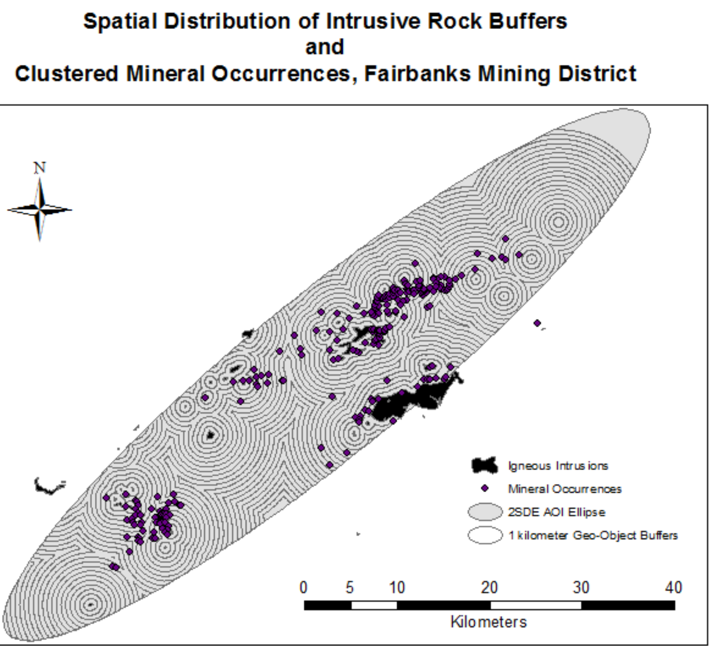


Figure 11. Spatial distribution of Fairbanks Mining District igneous intrusion 1 kilometer ring buffers and mineral occurrences.

**Exercize 4:**

An improved coincidence of a district wide mineral occurrence AOI with ring buffers associated with district wide igneous intursions can be achieved by creating a multi-part geo-barrier mask that confines the spatial distribution analysis to those parts of the district that have coincident sets of point-objects and geo-objects. An appropriat mask can be made by implementing a union of 2SDE spatial distribution ellipses calculated for the three mineral occurrence cluster seen in Figure 11.

Figure-12 illustrates three geographically separate clusters of mineral occurrences (identified as Ester, Gilmore, and Cleary Summit). A two standard deviation directional distribution ellipse polygon can be generatied for each mineral occurrence cluster using the ArcMap™ Statistical Analysis>Directional Distribution tool. The separate polygons then can be incorporated into a multi-part feature with the ArcMap™ Analysis>Union tool. Store the output from this union in the **Exercise_4** folder and name it "*CleryEsterGilmore_2SDE.shp.* "The resulting multi-part polygon, better defines a Fairbanks Mining District point-object AOI that is limited to areas of the district that contain both igneous intrusions and mineral occurrences. If used in the DDA tool as an optional *Constraining Barrier* parameter; the multi-part polygon will aid in enforcing the requirement that, for a valid spatial distribution analysis, the point-objects must span the occurrence of geo-objects within the defined spatial anlysis AOI.

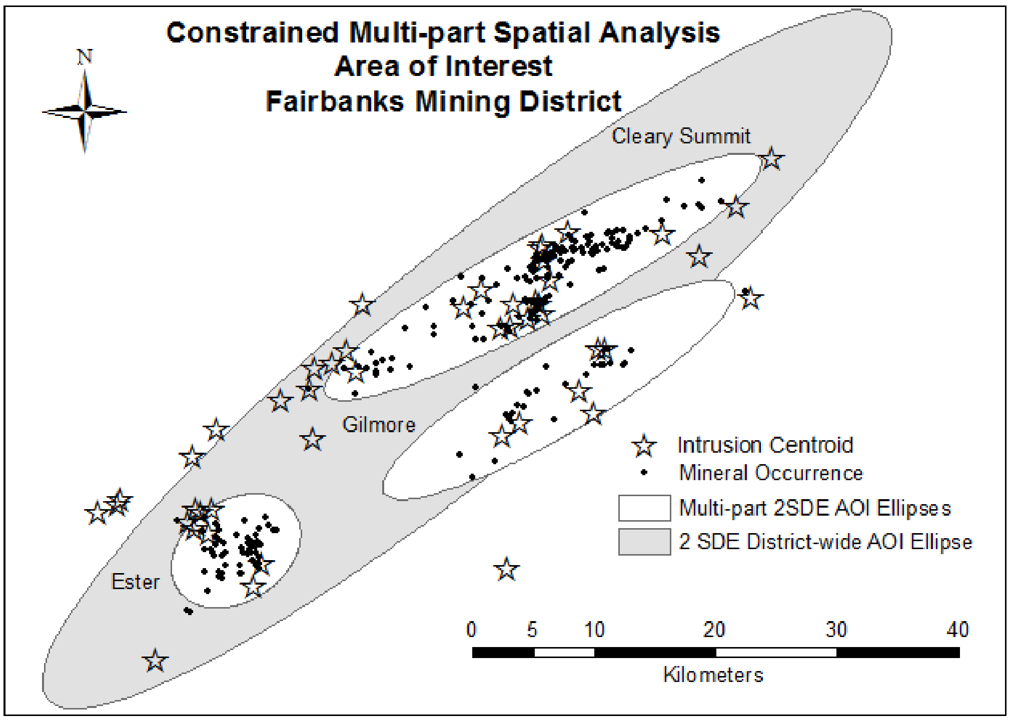


Figure 13. Clusters of mineral occurrences in the Cleary Summit, Gilmore and Ester areas of the greater Fairbanks Mining District are contained within a multi-part AOI comprising 2SDE point-object direction distribution ellipses. Only igneous intrusion ring buffers contained within the multi-point AOI will be incorporated in the district-wide mineral occurrence - igneous intrusion spatial distribution analysis.

Use the DDA tool with the following input parameters to conduct a spatial distribution analysis of the Fairbanks Mining District mineral occurrences relative to igneous intrusions within the district.

1. In the *Scratch Worskspace*: parameter window, browse to the newly created **Exercise_4** folder, and "Add" it.
2. In the *Point Objects*: parameter window, browse to the *FbkMinOccs.shp* file in the **SourcData** folder, and "Add" it.
3. In *the Geo-Objects*: parameter window, browse to the *FbkIgnInt.shp* file in the tutorial's **SourceData** folder and "Add" it.
4. In the *Buffer Distances*: multiple entries parameter window; enter a sequence of distances in units of 0.5 kilometers, from 0.5 to 5 kilometers.

5. In the *Area of Interest (AOI)* multiple selection parameter window, accept the default "2_STANDARD_DEVIATIONS" choice.
6. Do not select the *Buffer Distances measured in meters* parameter checkbox.
7. In the *Constraining Barrier (optional)* : parameter window, browse to the **Exercise_4** folder, Select> *ClearyGilmoreEster_2sde.shp* and "Add" it.
8. Click "OK" to run the DDA tool.

The final mineral occurrence multi-part AOI and included mineral occurrences and igneous intrusion ring buffers that are considered in Exercise 4 are displayed in Figure 14.

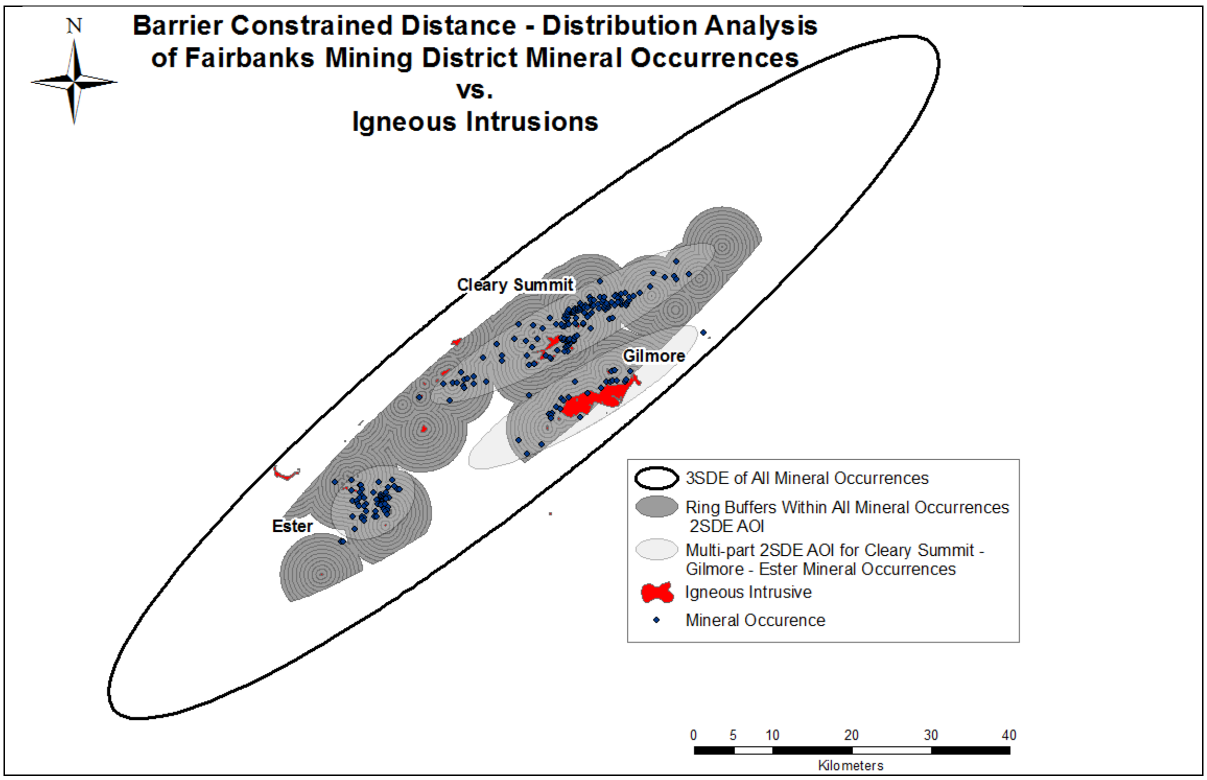


Figure 14. Primary elements employed in a multi-part barrier constrained distance distribution analysis of the Fairbanks Mining District mineral occurrences relative to igneous intrusive rocks. The multi-part AOI constraining barrier consists of the 2SDE directional distribution ellipses created for clusters of mineral occurrences in the Este, Gilmore, and Cleary Summit areas.

The DDA tool generates the Distance - Distribution and Beta - Distribution graphs shown in Figure 15 a.  and 15 b.

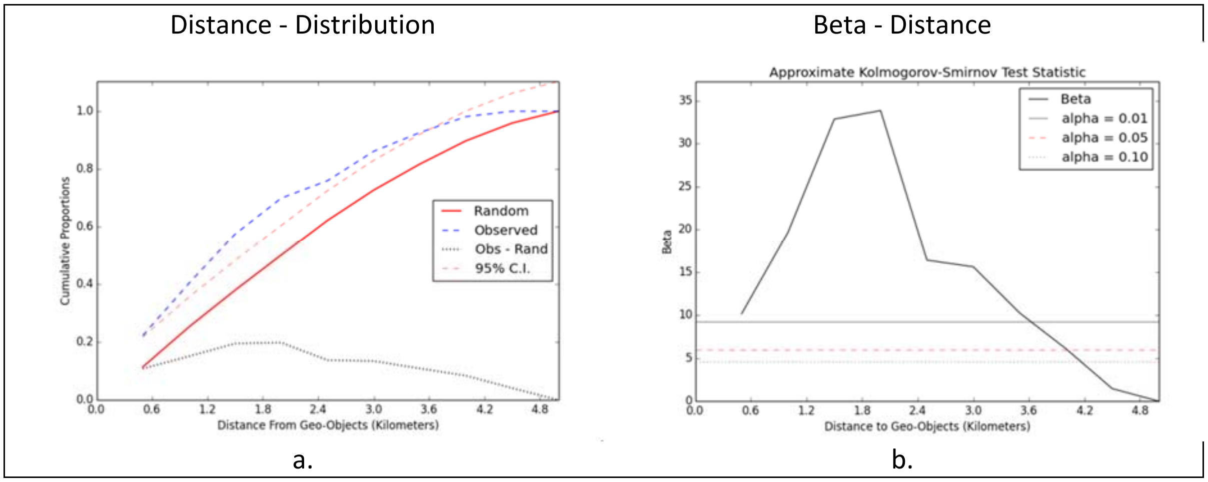


Figure 15 a. Distance - Distribution graph for 216 mineral occurrences in the Fairbanks Mining District relative to igneous intrusion ring buffers within the mineral occurrences' multi-part AOI; and b. Kolmogorov - Smirnov Beta statistic for a sequence of outer igneous intrusion ring buffer distances.

In Exercise 4, the buffering of geo-objects is halted at 5 kilometers because it is known from the results of Exercise_3 that no additional mineral occurrences are encountered beyond the 5 kilometer ring buffers. In Exercise 4, the 2SDE multi-part AOI does not include 4 mineral occurrence that were included in the single district wide 2 SDE AOI ellipse used in Exercise 3. Given the input parameters used in Exercises 3 and 4, one mineral occurrence is omitted from the spatial distribution analysis in both exercises. The final AOI considered for spatial distribution analysis in Exercise 4 is shown in Figure 14 as the intermediate gray-toned intersection of the multi-part constraining barrier and the igneous intrusion ring buffers. The AOI produced by including an optional constraining barrier more closely approximates Berman's (1986) requirement that the point-object locations span the geo-objects included in the spatial association analysis.

The Distance - Distribution graphs (Fig.s 14 a. and b.) indicate that there is a statistically significant positive spatial association between mineral occurrences and igneous intrusive contacts in the multi-part Fairbanks mining district AOI.  Note that the Beta values (Fig. 14 b.) are not inflated as in Exercise 3 (Fig. 10 b.). The plotted Exercise 4 data (Fig.14 a.) indicate with that the positive spatial association between igneous intrusive contacts and mineral occurrences seen within about 3.6 kilometers of intrusive contacts in the Fairbanks Mining District would happen by chance less than 1% of the time. The Distance - Distribution graph (Fig. 14 a.) and the data compiled in Table-5 also indicate that 98% of the mineral occurrences included in the final analysis are found within about 4 kilometers of an intrusive contact.

| FID | Distance | Buff_Area | Cum_Area | Ex | NumDeps | CumNumDeps | Ox | Ox_EX | UpConfi | Beta |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.5 | 45777027 | 45777027 | 0.114 | 48 | 48 | 0.222 | 0.108 | 0.217 | 10.14 |
| 1 | 1 | 55543773 | 101320800 | 0.252 | 39 | 87 | 0.403 | 0.151 | 0.355 | 19.63 |
| 2 | 1.5 | 51082949 | 152403750 | 0.379 | 37 | 124 | 0.574 | 0.195 | 0.482 | 32.84 |
| 3 | 2 | 49072298 | 201476048 | 0.501 | 27 | 151 | 0.699 | 0.198 | 0.604 | 33.84 |
| 4 | 2.5 | 48398659 | 249874707 | 0.622 | 13 | 164 | 0.759 | 0.138 | 0.725 | 16.38 |
| 5 | 3 | 42264480 | 292139187 | 0.727 | 22 | 186 | 0.861 | 0.134 | 0.830 | 15.61 |
| 6 | 3.5 | 36310655 | 328449843 | 0.817 | 14 | 200 | 0.926 | 0.109 | 0.920 | 10.24 |
| 7 | 4 | 32385336 | 360835178 | 0.898 | 12 | 212 | 0.981 | 0.084 | 1.001 | 6.08 |
| 8 | 4.5 | 24689574 | 385524752 | 0.959 | 4 | 216 | 1.000 | 0.041 | 1.062 | 1.45 |
| 9 | 5 | 16476353 | 402001105 | 1.000 | 0 | 216 | 1.000 | 0.000 | 1.103 | 0.00 |

Table 5. Cumulative frequency data derived from the distance - distribution analysis of Fairbanks mining district mineral occurrences relative to igneous intrusive contacts within the district.

Because the Beta statistic value calculated for the distance - distribution assessment cumulative frequency curves in the Fairbanks Mining District AOI (Fig. 14 b.) exceeds the $\alpha = .01$ significance level in the distance interval 0 - 3.6 km, the test indicates that the positive spatial association exhibited in Figure-14a. between observed mineral occurrences and igneous intrusive contacts within the Fairbanks mining district AOI is statistically significant at the $\alpha = .01$ significance level. Figure-14a. clearly displays the ring-buffer interval within which there is a significantly higher concentration of point-objects with respect to igneous contacts within the Fairbanks mining district AOI than would be expected to occur by chance. The positive spatial association of mineral occurrences with intrusive igneous contacts is greatest within 2 km of the contacts.

## Exercise 5:

Using the DDA tool, investigate the spatial distribution of all Fairbanks Mining District mineral occurrences relative to the district-wide faults (*FbkFaults.shp*). Use a constraining barrier multi-part polygon to ensure that Berman's (1986) requirement that the point-objects span the domain of the geo-objects within the AOI (Fig. 16).

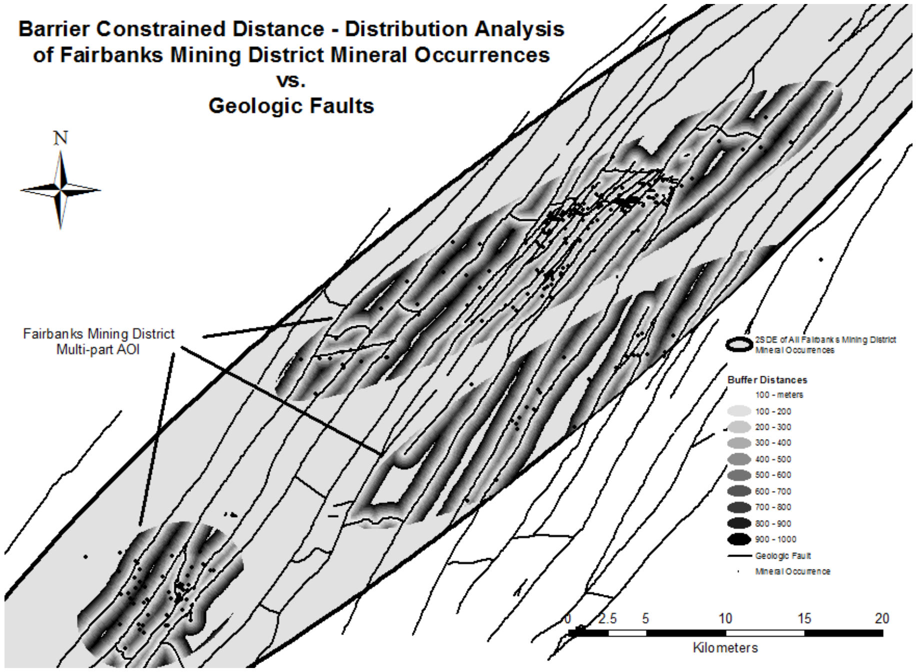


Figure 16. Multi-part barrier constrained distance - distribution analysis of all Fairbanks mining district mineral occurrences vs. mapped geologic faults.

The result of this analysis will indicate that the spatial association of mineral occurrences relative to faults in the multi-part AOI for the district as a whole are about the same as it is for the Cleary Summit area; that is, there is a statistically significant positive association and the optimal distance of mineral occurrences from a fault is 0 - 200 or 300 meters (Obs -Rand values are nearly equivalent at 200 and 300 meters). At those distances, about 60% - to - about 74%, respectively, of all known Fairbanks mining district mineral occurrences within the multi-part AOI are encountered. At 400 meters, about 82% of known mineral occurrences have been encountered (Fig. 17; Table-6).

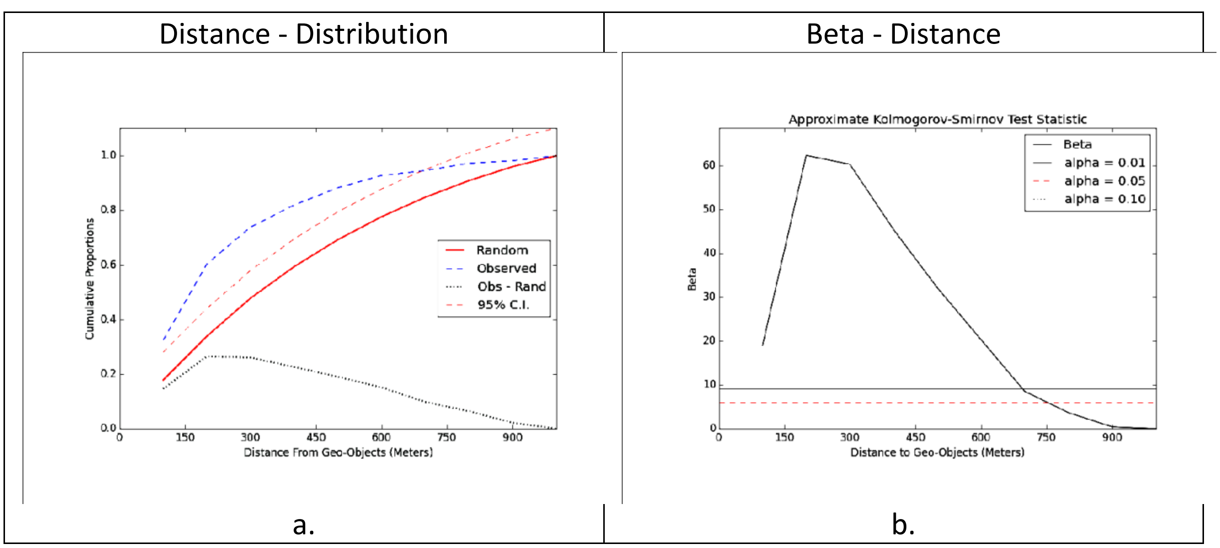


Figure 17 a. Distance - Distribution graph for 222 mineral occurrences in the Fairbanks mining district relative to the outer boundary of geologic fault ring-buffers within the district's multi-part mineral occurrences AOI; and b. Kolmogorov - Smirnov Beta statistic for a sequence of geologic fault ring-buffer distances within the district's multi-part mineral occurrences AOI.

| FID | Distance | Buff_Area | Cum_Area | Ex | NumDeps | CumNumDeps | Ox | Ox_EX | UpConfi | Beta |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 100 | 73565394 | 73565394 | 0.178 | 72 | 72 | 0.324 | 0.146 | 0.280 | 18.99 |
| 1 | 200 | 66348514 | 139913909 | 0.339 | 62 | 134 | 0.604 | 0.265 | 0.441 | 62.31 |
| 2 | 300 | 57621545 | 197535454 | 0.478 | 30 | 164 | 0.739 | 0.261 | 0.580 | 60.28 |
| 3 | 400 | 47738917 | 245274371 | 0.594 | 18 | 182 | 0.820 | 0.226 | 0.696 | 45.38 |
| 4 | 500 | 40861488 | 286135858 | 0.693 | 14 | 196 | 0.883 | 0.190 | 0.795 | 32.13 |
| 5 | 600 | 34728778 | 320864636 | 0.777 | 10 | 206 | 0.928 | 0.151 | 0.879 | 20.30 |
| 6 | 700 | 29474707 | 350339344 | 0.848 | 4 | 210 | 0.946 | 0.098 | 0.950 | 8.50 |
| 7 | 800 | 25091835 | 375431179 | 0.909 | 6 | 216 | 0.973 | 0.064 | 1.011 | 3.65 |
| 8 | 900 | 21290916 | 396722095 | 0.960 | 2 | 218 | 0.982 | 0.022 | 1.062 | 0.41 |
| 9 | 1000 | 16367103 | 413089198 | 1.000 | 4 | 222 | 1.000 | 0.000 | 1.102 | 0.00 |

Table 6. Cumulative frequency data for the spatial distribution of all mineral occurrences in the Fairbanks mining district relative to mapped geologic faults within the district.

Having demonstrated (Exercises 1 -5) there is a statistically significant positive spatial associations between mineral deposits and both geologic faults and igneous intrusions in the Fairbanks mining district, this information might be useful for focusing new exploration efforts in the district. One approach to creating exploration domains might be to intersect selected fault and igneous intrusion ring-buffers, guided by the data generated within the multi-part AOI used in Exercises 3 - 5.

Within the multi-part Fairbanks mining district's mineral occurrence AOI, about 98% of the district's mineral occurrences are found within 900 meters of a fault (Table-6), and about 98% of known mineral occurrences are within 4 kilometers of an igneous intrusive contact (Table-5). By intersecting the 0-900 meter area around faults with the 0-4 kilometer area around intrusive contacts, an "inclusive" multi-part polygon can be created that includes 90% of the known Fairbanks mining district mineral occurrences and 93% of those occurrences that have had some level of past mineral production (Fig. 18).

Within the multi-part Fairbanks mining district's mineral occurrence AOI, if the range for both fault and intrusive contact geo-object buffer distances are limited to the optimal distances (that is, 0-200 meters for the multi-part buffers surrounding faults and 0-2 kilometers for multi-part buffers surrounding igneous intrusions), these buffers can be intersected to form an "optimal" multi-part polygon that that delineates areas likely to have a much greater than random chance of mineral occurrences. This area includes 42% of known mineral occurrences and 40% of properties that have some level of past production in the Fairbanks mining district (Fig. 18).

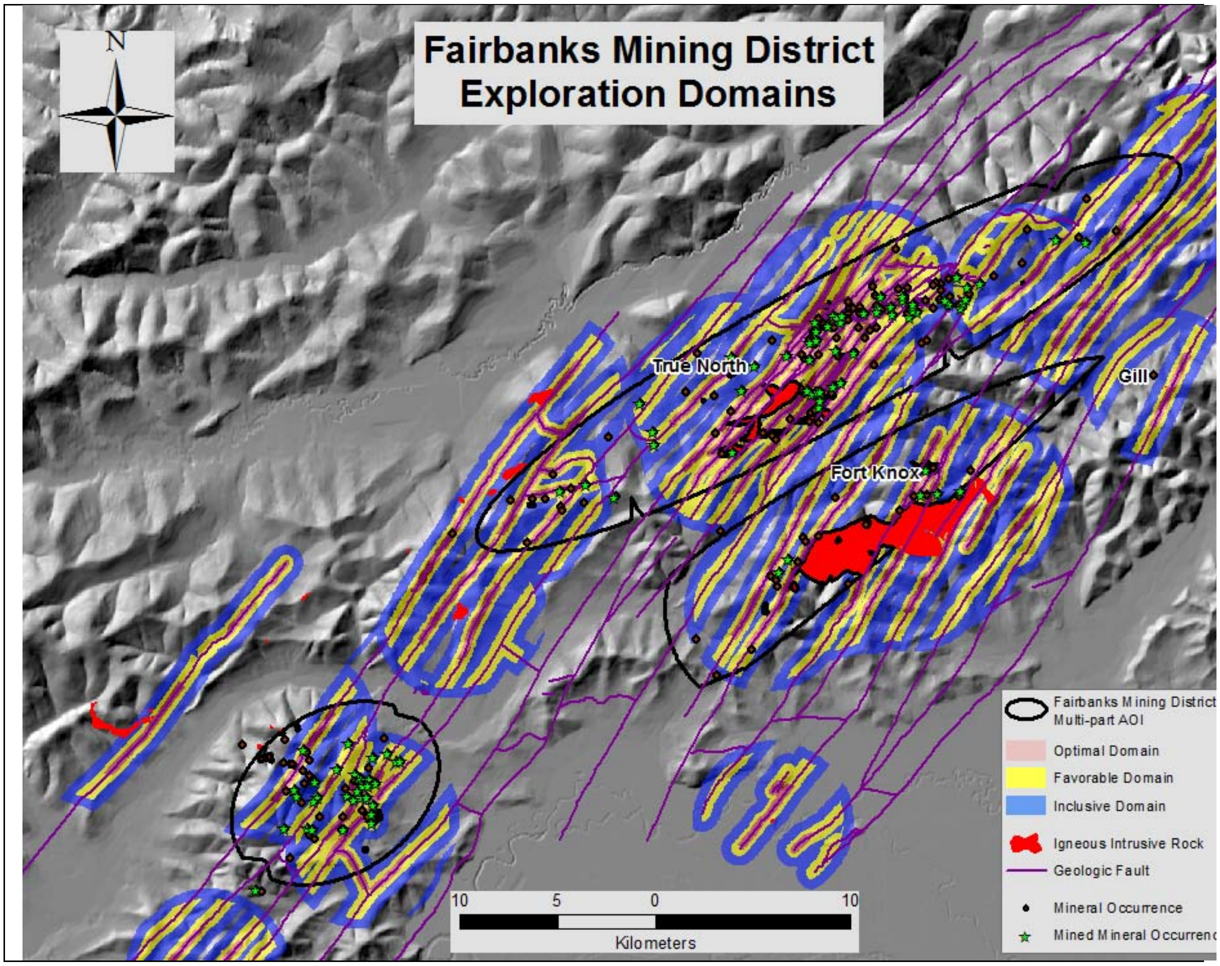


Figure 18. Fairbanks mining district mineral exploration domains defined by the outcome of a distance distribution analysis of mineral occurrences relative to district wide geologic faults and igneous intrusive contacts.

Within the multi-part Fairbanks mining district's mineral occurrence AOI, it also is reasonable to propose an intermediate "favorable" multi-part mineral exploration polygon that lies between the optimal- and inclusive-polygons defined above. Examination of the fault-associated cumulative frequency data in Exercise 4 and 5 suggests that a 3.5 kilometer buffer around all Fairbanks area igneous intrusives will include about 93% of all Fairbanks mining district mineral occurrences and that a 400 meter buffer around district faults will include 82% of Fairbanks mining district mineral occurrences. Within the multi-part Fairbanks mining district's mineral occurrence AOI, the intersection of Fairbanks mining district 400 meter fault buffers with 3. 5 kilometer igneous rock buffers results in a multi-part polygon that includes 74% of the district's mineral occurrences and 80% of properties that have some level of past production (Fig. 18).

Based on the results of the above exercises, if one planned additional exploration for mineral occurrences in the Fairbanks mining district, including areas outside the above multi-part AOI, serious consideration should be given to focusing efforts within the district-wide domains noted as "optimal," "favorable," or "inclusive" (Fig 18).

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