

An ArcGis Tool for the Fractal Analysis of Linear Geo-objects

Abstract

Applications for the concept of fractal dimensions in geology are still in a research and case study phase and are not routine. It is recognized that linear patterns that occur on geologic maps of fractures, faults and lithologic contacts are not true mathematical fractals. They do, however, have the fractal-like quality of being self-similar in appearance over a large range of geographic scale. Because of this characteristic, they are amenable to quantification with the use of a box counting algorithm that frequently has been employed to measure the fractal dimension of true fractal figures. As an acknowledgement that the values generated from the use of the box counting algorithm on map patterns of faults and contacts are not true fractal dimensions, the outcomes of box counting schemes applied to geologic features are sometimes referred to as a measure of geologic complexity rather than fractal dimensions. The Fractal Analysis tool that is the subject of this tutorial is based on a box counting algorithm implemented in the esri® ArcMap® software product.

Introduction

Hodkiewicz, et al. (2005) and Ford and Blenkinsop (2008) employed box counting algorithms, similar to those described below, to generate fractal dimension values in order to quantify the local geologic complexity of linear geo-features that occur on geologic maps of the Australia Yilgarn Craton and Mt. Isa Inlier, respectively. The target data for their box counting tools comprise digital regional geologic maps of faults and lithologic contacts.

Hodkiewicz, et al. (2005) was investigating whether there is a systematic spatial association of orogenic gold mineralization and geologic complexity in the Yilgarn Craton, Australia. Blenkinsop (2008) used a modified box counting technique to investigate the spatial relationship of copper mineralization to geologic complexity in Australia's Mt. Isa Inlier.

The geoprocessing tool described in this report provides a way to quickly conduct box counting processes within esri® ArcMap to generate point data that subsequently can be interpolated to produce a continuous raster map of local dimensions for fractal-like linear geo-features. The fractal-like dimension data are identified as **Fractal_D** values in the tool's final output shapefile. The point data are interpolated using the interpolators in ©ArcMap Spatial Analyst extension. The tool functions in esri® ©ArcMap v. 10.1 and 10.2.

The book *Fractals in the Earth Sciences* (Barton, C.C., and La Pointe, P.R., 1995) provides an introduction to some geologic uses for fractal geometry. A fractal is defined as: "a geometric

pattern that is repeated at ever smaller scales to produce irregular shapes and surfaces that cannot be represented in classical geometry," The Free Dictionary (2011). Benoit B. Mandelbrot (1982) summarized the observations and rules that described a new fractal geometry. Since then, mathematicians, scientists and engineers have worked to find ways to make practical use of Mandelbrot's insights. One avenue of investigation that has shown promise is to invert the process of creating Mandelbrot figures, that is., to start with a naturally occurring fractal-like pattern and determine its fractal dimension (D).

Below (Fig. 1) is a self-similar pattern of lines repeated at increasingly smaller scales. The constructed line pattern has fractal-like properties and resembles a digital tree. This figure was created by starting with a simple "Y" shaped line pattern of an initial size. Subsequently, the Y-shape was reduced to half the size of the original single line pattern; and the stem of the reduced Y-shapes were attached to the end of each of the original shape's branches at an angle to the branch that is equal to the angle of each initial branch to the stem of the initial Y-shape. This same algorithm was followed to add a third and fourth sublevel of Y-shapes to the figure.

The final constructed figure has fractal-like properties in that it is constructed of a single elemental shape replicated at decreasing sizes that follow a power law. The complexity of the figure varies significantly from a low level in the area near the initial "trunk" to a high level in the area near the outer "branches." Because this figure was constructed with replicated elements that follow a power law, the local complexity may be characterized by local fractal dimensions.

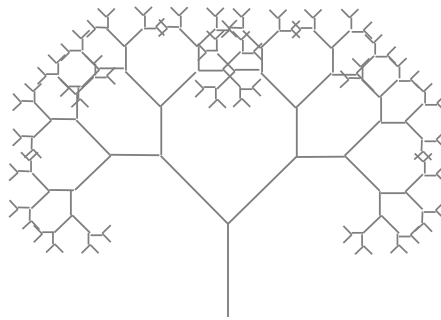


Figure 1. Simple self-similar linear pattern that exhibit a regular fractal-like geometry

In a similar manner, sets of rock fractures (fault traces and joints) and lithologic contacts are considered by some to have fractal-like properties. They do not fit the rigorous mathematical definition of a fractal (the patterns are not exactly self-similar, and the patterns do not persist to a scale approaching zero) but they often exhibit somewhat similar patterns over a wide range of scales from submillimeter to kilometers. For linear features, the fractal dimension of the spatial and scale distribution of the feature set can be approximated by using a process that imposes

aligned topological square polygon box grids having successively smaller square polygon box dimensions (Fig. 2) over the geo-feature set of interest and then counting the number of boxes in each grid that intersect a feature in the geo-feature set.

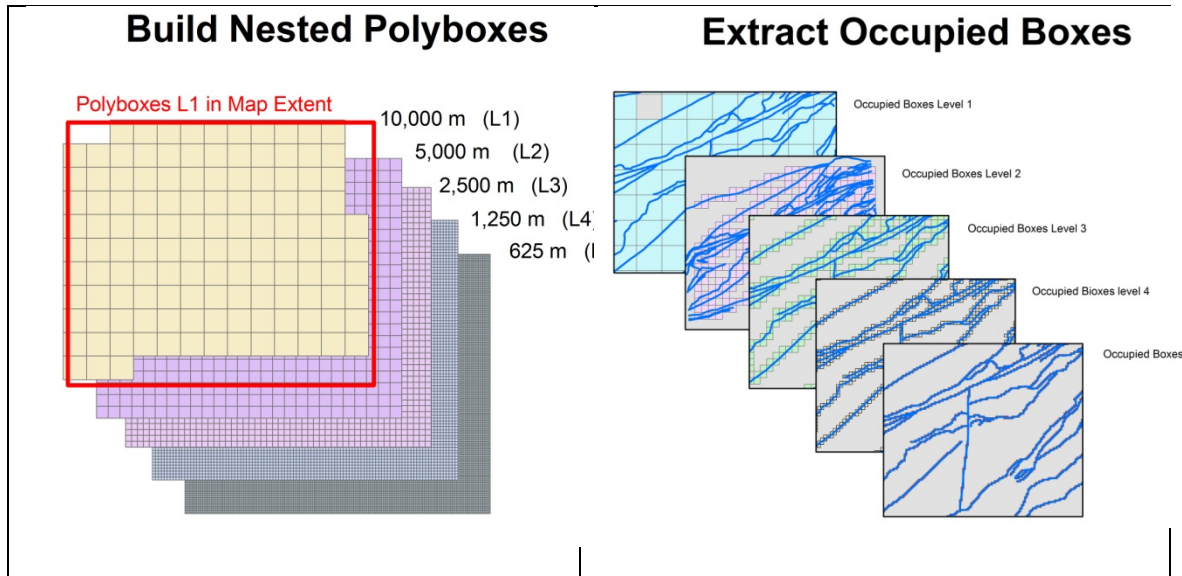


Figure 2. a set of polygon grids having successively smaller cell sizes are generated in ©ArcMap (a.) . These polygon cells ("boxes") are then intersected by the linear features of interest and all intersected cells at each grid level are counted (b.).

For each polygon box in the initial Level-1 polygon box grid, the relationship between the number of sublevel boxes occupied by a linear feature and the dimension of box side length at each sublevel is:

$$N_d \approx d^{-D} \text{ where: } \approx \text{ is used to indicate "proportional to"}$$

N is the number of grid cells, or boxes, intersected at each level

d is the side dimension of the grid cell, or box, at each level

D is defined as the Fractal Dimension

This can be written in Logarithmic form as:

$$\text{Log}(N_d) = -D (\text{Log } d)$$

Thus, for each Level-1 polygon box in the Level-1 polygon box grid: If the linear geologic feature set has the scale independent self-similar properties of a fractal pattern, a regression of the Logarithm of the count of sublevel boxes that intersect the features at each level vs. the Logarithm of the side dimension of the boxes at each level will plot as a straight line. The absolute value of the slope of that line approximates the fractal dimension of that portion of the feature set which is bounded by the Level-1 polygon box. These relationships are well illustrated for two large (Level-1) boxes in Figure 3 of Hodkeiwicz, et al., 2005.

Generating the data to calculate the local fractal dimensions of a geologic feature set is a computationally simple but tedious manual process. The following work flow diagram (Fig. 3) outlines the major steps needed to generate local fractal dimensions for a map of linear feature using the ©Fractal Analysis ver. 2 (Fractal Analysis)tool referenced in this tutorial.

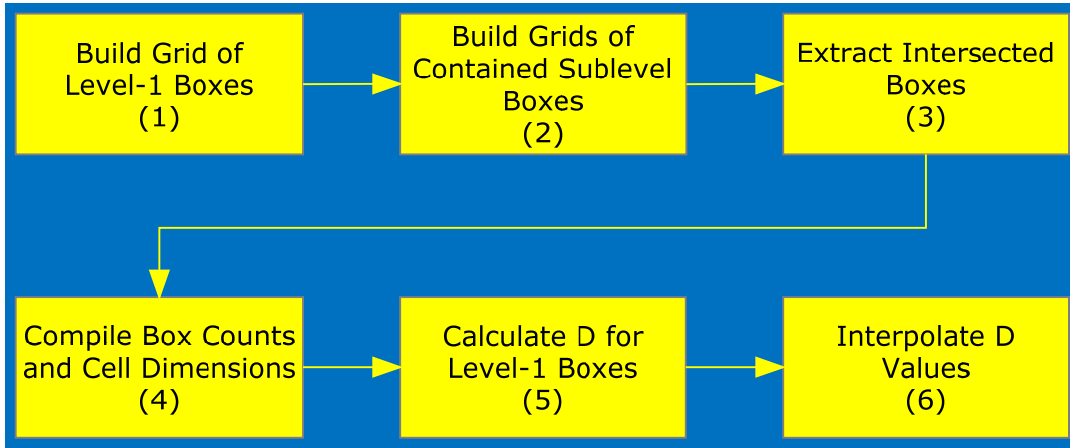


Figure 3. Diagrammatic process-flow for calculating local fractal dimensions for a regular grid of points distributed across a defined map area using esri® ©ArcMap, the ©Fractal Analysis ver. 2.0 tool, Wiltse (2014), and esri® Spatial Analyst interpolation tools.

1. Generate a regular grid of square polygon cells (boxes) that have a side dimension (d) that is the largest box size in the fractal analysis. These are the Level-1 boxes
2. Generate subgrids of square boxes having side dimensions that permit level-specific sets of subgrid boxes to nest perfectly within one of each of the Level-1 boxes.
3. For each Level-1 box, count the boxes from each set of contained subgrid boxes that are intersected by the features of interest.
4. Generate a data record for each Level-1 box that records the number of the contained intersected subgrid boxes and their side dimensions.
5. Using the values of box side dimensions and intersected-box counts compiled for each Level-1 box, calculate the slope of the least squares regression line derived from regressing the natural logarithms of the compiled data. For each Level-1 box, join the absolute value (D) of the slope value to the centroid of the box.
6. Use ©ArcMap® Spatial Analyst interpolation tools to generate a raster of local fractal dimension estimates for all points within the limits of Level-1 intersected boxes.

Any box counting process has an inherent spatial resolution fixed by the dimensions of the largest (Level-1) box used to accumulate the counting data of the sublevel boxes. If, for example, the size of the boxes in the first Level-1 box grid is 10 km x 10 km, then the resulting D-values joined to box centroids will each represent the fractal dimension for an associated 10^2 km area.

Because of the way in which the box counting algorithm functions, one cannot arbitrarily decrease the Level-1 box size to improve spatial resolution. In practice, the spatial resolution of the Level-1 box grid is limited by the necessity of choosing a box dimension large enough so that each level 1 box will be intersected at least once by a linear geo-feature of interest. This guidance is not absolute, but if one hopes to interpolate a useful raster map of local fractal dimensions, then a nearly contiguous grid of Level-1 box centroids is required.

If Level-1 box dimensions are quite large the subsequent raster map will not discriminate subtle changes in local fractal dimensions. A coarse Level-1 spatial resolution may also result in a large offset of local minimum and maximum D-values in areas of sparse linear geo-features. To some extent, these shortcomings in the box counting algorithm can be ameliorated by modifying the basic box counting tool so that it incrementally translates the box counting grids to new positions on the map to generate additional D-value data. These new data points can then be incorporated into a single comprehensive dataset prior to interpolation to a raster map of local fractal dimensions. The data points in the comprehensive data set represent the fractal dimensions of overlapping Level-1 box areas.

In anticipation of the need to quantify the geologic complexity of fractal-like geo-features in a variety of settings, the Fractal Analysis tool allows the user to vary the spatial density of the output shapefiles Fractal-D data points.

In general, higher density of data points results in better resolution of local variations in the Fractal-D values of the mapped geo-features. Higher density data-point arrays, however, take longer to generate

After the Fractal Analysis tool has created a set of data points, the ©ArcMap Spatial Analyst interpolation tools are used to create a continuous raster image of the source map's local fractal dimensions. For the Livengood data used in this tutorial, the Natural Neighbor and Inverse Distance Weight interpolators generate rasters that closely conform to the Fractal Analysis tool's calculated data point Fractal-D values. In general, the Inverse Distance Weight interpolator provides a smoother surface when used with higher density data point arrays.

Tutorial

The **Fractal Analysis Tool** requires the esri® ©ArcMap Spatial Analyst extension and functions in ArcGis® Desktop v. 10.1 and 10.2. The Python script for the **Fractal Analysis Tool** was written

with Python version 2.7 which is included with ArcGis® v. 10.x The **Fractal Analysis Tool** was developed on a desktop computer having a 64-bit Windows 7 operating system.

The following tutorial exercises are provided to illustrate how to use the Fractal Analysis tool to generate data that can be interpolated to a raster map of the local fractal dimensions associated with a set of regional faults. For this tutorial, shapefiles of the Livengood, Alaska 1:250,000-scale Quadrangle boundary and mapped geologic faults within the Livengood Quadrangle are provided. These shapefiles, along with the **Fractal Analysis ver. 2.0** tool in the **Fractal Analysis Tool.tbx** and the tool's Python script are included in the subfolders of the **Fractal Analysis** folder. The **Fractal Analysis Tool** is a "script" tool and will be displayed as such when it is displayed in the ©ArcMap Catalog window.

Installing the Fractal Analysis Tool:

This tutorial requires the files contained in the **Fractal Analysis Tutorial** subfolders: **Fractal Analysis Toolbox** and **Fractal Analysis Data**.

Installing the **Box Count Fractal Analysis.tbx** and the tutorial data files is accomplished by following the workflow outlined below:

1. From the disk or download file provided, copy the **Fractal Analysis Tutorial** folder to any of your disk drives that can be accessed by ©ArcMap. The Fractal Analysis Folder contains sub-folders for:
 - Documentation,
 - Fractal Analysis Toolbox,
 - Scratch and Results folders,
 - Source Data.
2. Link the Python script, *MultiTrans.py*, found in the **Fractal Analysis Toolbox** folder. This is accomplished in ©ArcMap by browsing to the tool in the Catalog window of ©ArcMap:

Select **Fractal Analysis Tutorial > Fractal Analysis Toolbox > Box Count Fractal Analysis.tbx>Fractal Analysis ver 2.0**. Right-click the *Fractal Analysis ver 2.0* script tool, Select "Properties>Source Tab", and browse to the *MultiTrans4.py* file located in the **Fractal Analysis Toolbox** folder, select that file and then Select OK at the bottom of the Properties dialog box.

Applying the Fractal Analysis Tool:

- Set the ©ArcMap session Environments Workspace to: **<Drive>:\Fractal Analysis Tutorial\Scratch>Results>**

- Double-click the *Fractal Analysis ver 2.0* tool to open it. The following dialog window will appear (Fig. 4). The dialog window prompts for 9 user-specified tool parameters.

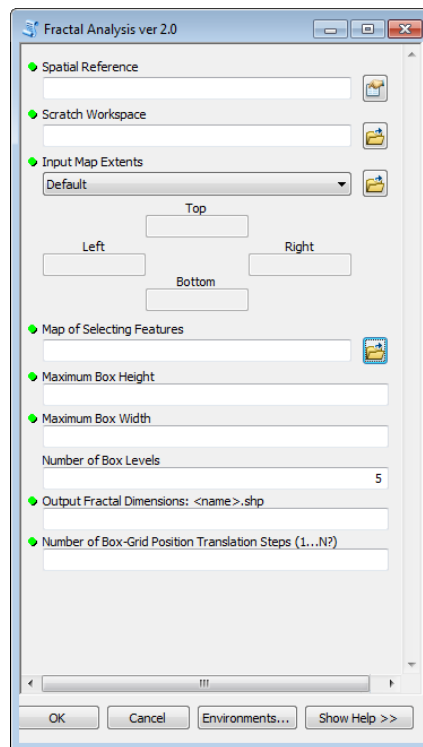


Figure 4. Fractal Analysis Tool input parameter dialog interface window

1. Spatial Reference: In the Spatial Reference window, browse to *Projected Coordinate Systems>Continental>North America>Alaska Albers Equal Area Conic>OK*.
(The *Fractal Analysis ver 2.0* tool requires the use of a projected coordinate system with distance measures expressed in meters.)
2. Scratch Workspace: Browse to: **<Drive>:\Fractal Analysis Tutorial\Scratch**; Enter **Scratch** folder name in the Scratch Workspace (Name :) dialog window and click "Add." The *Fractal Analysis ver 2.0* tool generate many intermediate files. They will all be written to this scratch workspace. They may be deleted after the tool has completed its processing.
3. Map Extents: Browse to **<Drive>:\Fractal Analysis Tutorial\Source Data\Maps**; Enter *LivQuad.shp* in the (Name :) dialog window and click "Add."
4. Map of Selecting Features: Browse to **<Drive>:\Fractal Analysis Tutorial\Source Data\Maps**; Enter *LGFaultsAlb83.shp* in the (Name :) dialog window and click "Add."
5. Maximum Box Height: Enter 100000 in the Maximum Box Height window of the Fractal Analysis ver 2.0 dialog box.

6. Maximum Box Width: Enter 100000 in the Maximum Box Width window of the Fractal Analysis ver 2.0 dialog box.
7. Number of Box Levels: The default value for this parameter is 5. Accept this value and proceed to the next parameter window. This parameter determines how many sublevel grids of smaller boxes are generated. A value of 5 will produce four ordered pairs of $\ln(d)$ and $\ln(\text{count})$ data for each Level-1 box that intersects a geo-feature. These are the inputs for the regression equation that will generate the Fractal-D value for the associated Level-1 box.
8. Output Fractal Dimensions: <name>.shp: Enter the name you would like to use for the output data point array shapefile. Be sure to include the .shp suffix or the tool will abort. This file will be written to the current ©ArcMap session's Workspace.
9. Sqrt (Number of Box-Grid Position Translation Steps) [1...N?]: The spatial resolution of variations in the local fractal dimensions of a map area can be improved by generating a higher density of box-count data points. This can be achieved by translating the box-grids across the map of geo-features in a systematic manner. The number entered in this parameter window determines how many box-grid translations will be made. Enter the value 2 for this parameter.
10. When all the input parameters have all been entered into the Fractal Analysis Tool dialog window, Select **OK**. The time it takes the Fractal Analysis tool to calculate the local fractal dimension data points varies with the size of the area of interest, Level-1 box dimensions, number of sublevels, and number of box-grid translations. With the parameter values suggested above, the tool will take about 5-6 minutes to process the input data.

Additional Parameter Notes

Spatial Reference: refers to the geographic projection of the ArcGis® shapefiles that are used to define the area of interest and selecting features included in the fractal analysis, e.g., "Alaska Albers Equal Area Conic." The spatial reference can be entered by using the Browse button, located to the right of the *Spatial Reference* parameter input window, to navigate to the Projected Coordinate Systems in the ArcGis® Spatial Reference Properties dialog window. Continue navigating through the available coordinate systems to select the projected system that matches that of the input data shapefiles. For this tutorial the spatial reference is: *Projected Coordinate Systems>Continental>North America>Alaska Albers Equal Area Conic*.

Scratch Workspace: refers to the full path specification that includes the folder name in which the user would like to store the intermediate files generated by the **Fractal Analysis Tool**. It generally is convenient to store the generated files in a scratch folder within the current project folder. If a scratch workspace folder already exists, the Browse button located to the right of the *Scratch Workspace* parameter window can be used to navigate to the project scratch workspace folder, and select it. All Fractal Analysis Tool intermediate products will be sent to the selected folder. The final Fractal-D data point shape file will be sent to the Workspace currently specified in the ©ArcMap Geoprocessing Environments settings.

Input Map Extents: Map extents may be entered in a number of ways as specified by the drop down pick list embedded in the *Input Map Extents parameter window*. A common way to enter map extents is to use the browse button to open a dialog window that allows navigation to the project shapefile used to bind the fractal analysis area of interest. For this tutorial that would be the Livengood Quadrangle boundary shapefile, "*LivQuad.shp*." Selecting that shapefile and then selecting the "Open" button in the dialog window will automatically enter the extents of the *LivQuad.shp* file into the appropriate parameter windows in the **Fractal Analysis Tool** dialog interface window. Alternatively one can choose to explicitly type the bounding extent values directly into the 4 extent parameter entry windows of the *Input Map Extents* area of the **Fractal Analysis Tool** dialog window.

Map of Selecting Features: refers to the shapefile that contains the linear features which comprise those features that will be analyzed by the **Fractal Analysis Tool**. The map of selecting features shapefile for the tutorial exercise is "*LGFaultsAlb83.shp*" and can be selected by navigating to it via the browse button located to the right of the *Map of Selecting Features* parameter window.

Maximum Box Height: is the dimension, in meters, of the Level-1 box side length. In general this value should be large enough so that nearly every Level-1 box will intersect at least one feature of interest. For the Livengood fault map 10000 meters meets this criterion fairly well. Enter the appropriate value in the *Maximum Box Height* parameter window.

Maximum Box Width: Because the Fractal Analysis Tool is designed to work with square boxes, the maximum box width should be the same as the maximum box height. Enter the appropriate value in the *Maximum Box Width* parameter window.

Number of Box Levels: The number of box levels should be at least 4. This number may be estimated by considering what minimum box dimension is likely to result in a significant number of sublevel boxes that intersect more than one feature of interest in areas of feature concentration. If this number is made too large, the relationship between box counts and box dimension will cease to conform to a mathematical power law and the absolute value of the slope of the regression curve for the equation:

$$\text{Log}(N_d) = -D (\text{Log } d)$$

will not closely approximate the fractal dimension (D). For this exercise, enter the value (5) in the *Number of Box Levels* parameter window.

Output Fractal Dimension Shapefile: Enter the name of the shapefile that will contain the fractal dimension values for each of the Level-1 boxes. Each record in the attribute table of this file contains the *Point_X* and *Point_Y* coordinates for the centroid of the Level-1 box associated with the record. Each record also includes the sublevel box counts and box side dimensions for feature-intersected sublevel boxes contained within the associated Level-1 box, and the fractal dimension (*Fractal_D*) of the associated Level-1 box. These values can be extracted to an external program to independently plot the record's data points and calculate the regression line if one wishes to visually examine the fit of the least squares regression for an individual Level-1 box.

sqrt (Number of Box-Grid Position Translation Steps [1...N?]): Because of the way in which the grids are translated in both the W-E and N-S directions the total number of translation steps is the square of the number entered.

Generating a raster map of local fractal dimensions:

A regional raster map representing the local fractal dimensions associated with the features of interest is readily made using the interpolation tools located in the ArcGis® Spatial Analyst toolbox. A Natural Neighbor interpolation raster map of local fractal dimensions for the Livengood Quadrangle faults is shown in Figure 5. The interpolation was made from the output data point array produced from the data and input parameters specified above. Increasing the box-grid translation parameter to 5 results in the raster map of local fractal dimensions shown in Figure 6. The map in Figure 6 was made with the Inverse Distance Weight interpolator using a fixed search distance of 3000 m and the inverse distance weight factor set to .9.

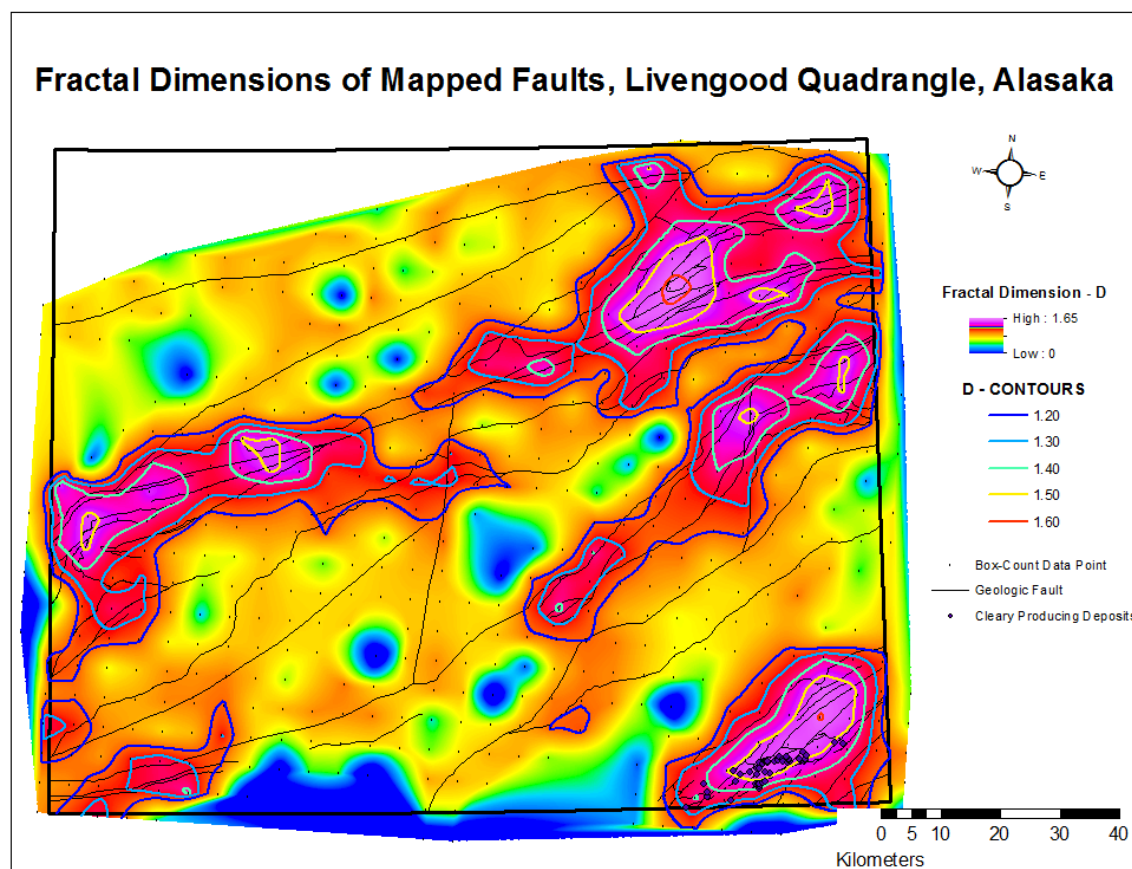


Figure 5. Natural Neighbor interpolation of local fractal dimensions of Livengood Quadrangle geologic faults, calculated from data point values compiled on Level-1 box center points located 5 kilometers apart.

Fractal Dimensions of Mapped Faults, Livengood Quadrangle, Alaska

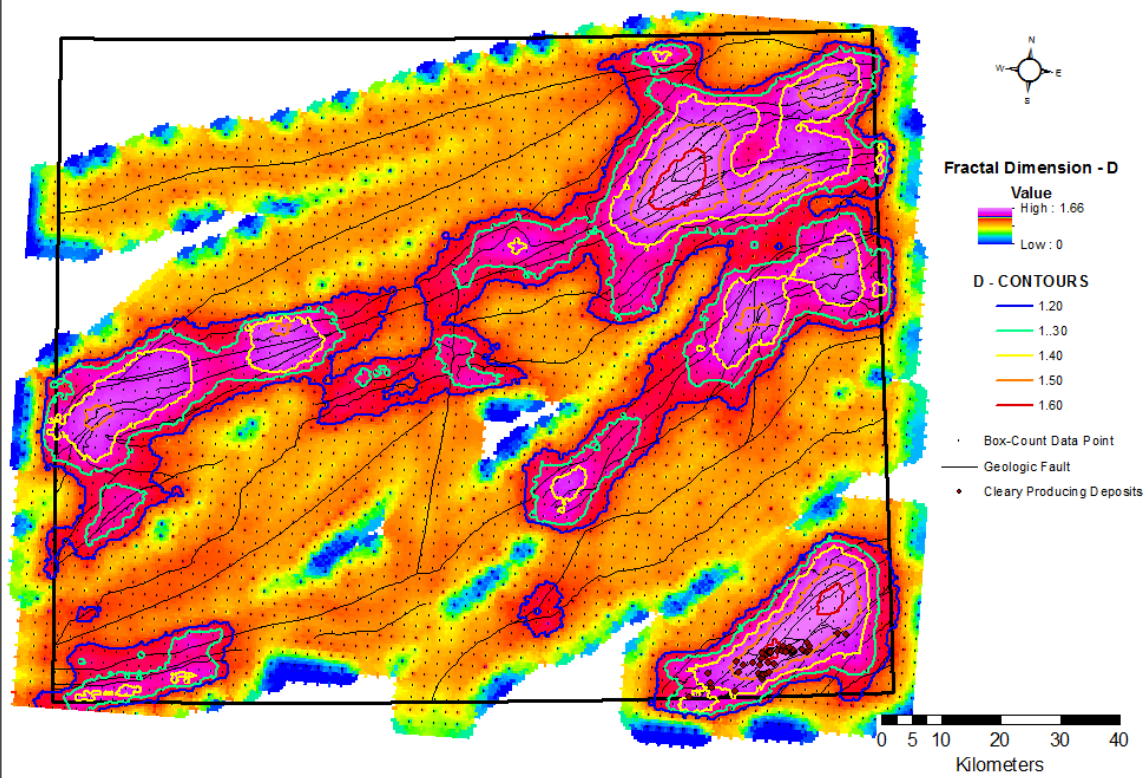


Figure 6. Inverse Distance Weight interpolation of local fractal dimensions of Livengood Quadrangle geologic faults, calculated from data point values compiled on Level-1 box center points located 2 kilometers apart.

Discussion

As an exercise, one can use the tutorial dataset and *Fractal Analysis* tool to generate fractal-D data points using 1, 3, or 4 as the entry values for the sqrt (Number of Box-Grid Position Translation Steps) parameter window. It will be apparent that fewer grid position translations result in a more generalized raster interpolation of the local fractal dimensions associated with the target geo-features. The basic configuration of the variation in local fractal dimensions, however, emerges even when interpolating a low density data point array. In the case of the Livengood fault data, it also will be apparent that the maximum fractal dimension values do not vary much with a change in the density of points generated by the *Fractal Analysis* tool.

Decreasing the dimensions of the Level-1 boxes, increasing the number of sublevels, or increasing the number of translations will increase the time required for the fractal analysis tool to complete its data processing for a given geographic area. The tool requires about one and a half minutes to process the Livengood data when 10 km x 10 km Level-1 boxes are specified with 5 levels of box-grids and one translation of the grid set. When a value of 2 is entered in the Box-

grid Transition Steps parameter window, the *Fractal Analysis* tool require about 5 minutes to process the data. Figure 6 has a Fractal-D data point array of 3632 data points. It required about 40 minutes to generate.

When initiating and investigation of the local variation of fractal dimensions of a new set of data, one may want to start with parameter settings that will process the data quickly in order to determine Level-1 box dimensions and the number of sublevels to specify. In general, Level-1 box dimensions that are the minimum that results in achieving a continuous coverage of the area of interest with boxes that have intersected at least on geo-feature of interest, work well in the geoprocessing workflow outlined in this tutorial.

The number of box levels specified should be at least 4 to achieve 3 data points in the regression of $\ln(\text{counts})$ vs. $\ln(d)$. One can specify too many sublevels. If the sublevel boxes become too small, they just trace individual features linear features and create a "roll-off" effect in a plot of the data. Roll-off distorts the slope of the $\ln(\text{count})$ vs. $\ln(d)$ regression, and thus the reported Fractal_D (fractal dimension) value.

References:

- Barton, C.C, and La Pointe, P.R., 1995, *Fractals in the Earth Sciences*, Plenum Press, New York.
- Ford, A., and Blenkinsop, T.G., 2008, Evaluating geological complexity and complexity gradients as controls on copper mineralization, Mt Isa Inlier, Australian Journal of Earth Sciences, v. 55, pp. 13-23.
- Hodkiewicz, P.F., Weinberg, R.F., Gardoll, S.J., and Groves, D.I., 2005, Complexity gradients in the Yilgarn Craton: fundamental controls of crustal-scale fluid flow and the formation of world-class orogenic-gold deposits, Australian Journal of Earth Sciences, v.52, pp. 831-841.
- Mandelbrot, B.B., 1982, *The Fractal Geometry of Nature*, Freeman, San Francisco.
- The Free Dictionary, 2011, <http://www.thefreedictionary.com/fractal> , Accessed March 16, 2011.

